Mathematical Physics 1: Linear Algebra, CMI

Problem set 8

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Due at the beginning of class on Tuesday, September 1.

Projections, Orthogonal matrices

1. Let $A = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Is A anti-symmetric? Why?
2. Find A^n for all $n = 0, 1, 2, \cdots$. (Hint: the answer is very simple, A^n is periodic in n.)
3. Define the matrix exponential for any real number x, as the matrix $e^{Ax} = \sum_{n=0}^{\infty} \frac{A^n x^n}{n!}$. Obtain a formula for e^{Ax} as a linear combination $e^{Ax} = f(x)I + g(x)A$. Find $f(x), g(x)$.
4. Using the above-obtained formula, find whether e^{Ax} is an orthogonal matrix.
5. Suppose the measured currents flowing through a circuit element for the following applied potential differences are $\begin{pmatrix} V: & -3 & -1 & \frac{1}{2} & 1 & 2 & 3 \\ I: & -2 & -1.5 & 0.1 & 2 & 2.5 & 3 \end{pmatrix}$. Using the least squares method, you will fit a straight line $I = GV + C$ through the data. Draw a rough figure of the data on an $I - V$ plot and indicate the expected straight line and the errors that are to be minimized.
6. Find the normal equations explicitly.
7. Solve the normal equations either by Gaussian elimination or matrix inversion.
8. What is the best-fit value of resistance?
9. What is the best-fit value of current in the absence of any applied voltage?
10. The above best-fit line through the data minimizes the norm^2 of the error vector $||e(x)||^2 = ||b - Ax||^2$ where the overdetermined system was $Ax = b$. Obtain the value of this minimal error norm $||e(x)||_{min}$ in the above example. First get a formula for it and then its numerical value.