MP1 for BP1: Linear Algebra, CMI

Problem set 1

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Write solutions and hand in at the beginning of class on friday 7 august. You are free to discuss with your classmates, but write the solutions yourself. Be brief!

Consider the pair of equations in two unknowns

$$x - 2y = 1, \quad 2x - 4y = 8 \tag{1}$$

1. Write the system of equations as a single matrix equation $Ax = b$,
2. What are A, b and their sizes?
3. What is the domain space?
4. What is the target space?
5. Plot the row picture, i.e. domain space picture (Hint: the equations are those of a pair of lines). Why is it called domain space picture?
6. What are the solutions and how many are there?
7. Explain the solutions in terms of the picture.
8. Re-write the equations as a 'linear combination' of columns.
9. Plot the column picture, i.e., target space picture. Why is it called target space picture?
10. Explain the solutions in terms of the column picture.
11. What is the range or image of A? i.e. what vectors are in the image?
12. Draw a picture to show that the image is contained in the target.
13. What is the determinant of A? (If you don't know what the determinant is, look it up!)
14. Think of A as a map from the domain to the image. Can you invert it?
15. How must the right side of the second equation be changed to get infinitely many solutions? What are they?