Mathematical Methods, Spring 2024 CMI
Assignment 8
Due by the beginning of the class (1030 am) on Tue, Mar 5, 2024
Volume form, Jacobi identity

1. $\langle \mathbf{4} \rangle$ Find the Riemannian volume form on the 2-torus arising from the induced metric on T^2 from its embedding $(\theta, \varphi) \mapsto ((R + r\cos\theta)\cos\varphi, (R + r\cos\theta)\sin\varphi, r\sin\theta)$ in 3d Euclidean space. Hint: You may use results from earlier assignments.

2. $\langle \mathbf{10} \rangle$ Show that the Jacobi identity $\{f, \{g, h\}\} + \{h, \{f, g\}\} + \{g, \{h, f\}\} = 0$ for any three functions f, g and h on phase space may be written as the following quadratic condition on the Poisson tensor:
$$r^{il}\partial_l r^{jk} + r^{kl}\partial_l r^{ij} + r^{jl}\partial_l r^{ki} = 0. \tag{1}$$
Hint: Start with $\{g, h\} = r^{ij}\partial_i g \partial_j h$ and use it repeatedly. You should find that all terms involving second derivatives of f, g and h cancel out due to antisymmetry of r.

3. $\langle \mathbf{4} \rangle$ Show that the inverse of an invertible antisymmetric matrix is antisymmetric, i.e., if $r\omega = I$ and $r^t = -r$ then $\omega^t = -\omega$.