

## Mathematical Methods, Spring 2024 CMI

### Assignment 2

Due by the beginning of the class (1030 am) on Jan 23, 2024

Jacobian matrix, diffeomorphism, multiply connected manifold

1.  $\langle 2 + 4 \rangle$  (a) Find the **Jacobian matrix**  $J$  for the transformation from plane polar  $(r, \phi)$  to Cartesian  $(x, y)$  coordinates on the punctured plane  $[(x, y) \neq (0, 0)]$ . Find  $\det J$  and show that  $J$  is invertible on the punctured plane. (b) Find  $J^{-1}$  and compare it with the Jacobian matrix for the inverse transformation.
2.  $\langle 7 \rangle$  Propose a **diffeomorphism**  $\phi(x)$  between the open interval  $(0, 1)$  and the real line  $\mathbb{R}$ , using a rational function. Explain why it qualifies as a diffeomorphism.
3.  $\langle 2 + 2 + 4 \rangle$  **Multiply connected space.** An infinite right-circular cylinder  $S^1 \times \mathbb{R}$  is connected but not simply connected. (a) Given a point  $p$  on the cylinder, draw a closed curve that begins and ends at  $p$  that is homotopic to a point. (b) Draw and describe a closed directed curve on the cylinder beginning and ending at  $p$  which cannot be continuously shrunk to  $p$ . (c) Classify (via figures and a brief explanation) all closed curves based at  $p$  into distinct homotopy classes (equivalence classes under continuous deformation) using an appropriate integer with a suitable name.