Mathematical Methods, Spring 2024 CMI

Assignment 14

Due by 1030 am on Monday, May 6, 2024

Orthogonal matrices, Lie algebras

1. $\langle \mathbf{3}+\mathbf{3}+\mathbf{3} \rangle$ Find these 3×3 orthogonal matrices, verify they satisfy $A^t A = I$ and find $\det A$. (a) A counterclockwise rotation by angle θ about the z axis. (b) A reflection in the xy, yz and zx planes. (c) A reflection through the origin.

2. $\langle \mathbf{6} \rangle$ Structure constants of $O(3)$. Suppose we take e_1, e_2, e_3 with matrix elements $(e_i)_{jk} = -\epsilon_{ijk}$ as a basis for $O(3)$. Show that their commutators may be expressed as $[e_i, e_j] = \epsilon_{ijk} e_k$.

3. $\langle \mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{3} \rangle$ Characterize the matrices that lie in the Lie algebras of the following matrix groups and use this to find the real dimensions of the corresponding Lie groups: (a) the real general linear group $GL_n(\mathbb{R})$, (b) the special linear group $SL_n(\mathbb{R})$ which is the subgroup with unit determinant and (c) the orthogonal group $O(n)$, (d) $SO(n)$.