

## Mathematical Methods, Spring 2024 CMI

### Assignment 13

Due by the beginning of the class (1030 am) on Tue, Apr 30, 2024

Direct/semidirect product, symmetric group

1. **⟨4+3⟩** (a) Find the composition law for the direct (Cartesian) product group  $G = C_2 \times C_2$ . Express it in terms of a multiplication table. (b) Is  $G$  isomorphic to the cyclic group  $C_4$ ? Why?
2. **⟨5⟩** Suppose  $G = H \rtimes N$  is the semidirect product of the group  $H$  acting via  $\varphi$  on  $N$ . Recall that the composition law is  $(h, n) \cdot (h', n') = (hh', n\varphi_h(n'))$ . Show that  $N$  may naturally be regarded as a normal subgroup of  $G$  in the sense that the set of elements  $(e_H, n)$  forms a normal subgroup of  $H \rtimes N$  isomorphic to  $N$ .
3. **⟨6⟩** Show that any group  $G$  of order  $n$  may be realized as a subgroup of the **permutation group**  $S_n$ . Hint. Use left multiplication  $L_g$  to associate with any element  $g \in G$  a permutation of the group elements. Show that it is a homomorphism.