Mathematical Methods, Spring 2024 CMI
Assignment 11
Due by the beginning of the class (1030 am) on Tue, Apr 2, 2024
subgroup, conjugation, homomorphism, abelian/nonabelian

1. $\langle 4 \rangle$ Show that H is a **subgroup** of G if and only if given $h, k \in H$, $hk^{-1} \in H$.

2. $\langle 4 \rangle$ If H is a subgroup of G, then verify that $H_g = gHg^{-1}$ is also a subgroup. It is called the **conjugate subgroup** of H by the element g.

3. $\langle 4 \rangle$ Suppose $\phi : G \to G'$ is a homomorphism of groups. Show that the **homomorphic image** $\phi(G) \subset G'$ is a subgroup of G'.

4. $\langle 4 \rangle$ Find an **isomorphism** between the additive group of real numbers $(\mathbb{R}, +)$ and the multiplicative group of strictly positive real numbers $(\mathbb{R}^+, \times)$.

5. $\langle 4 \rangle$ Show that the group of 2×2 orthogonal matrices $O(2)$ is **nonabelian**.