Mathematical Methods, Spring 2024 CMI

Assignment 10

Due by the beginning of the class (1030 am) on Tue, Mar 19, 2024

Covariant derivative, curvature

1. $\langle \mathbf{5} \rangle$ Show that the metric tensor is covariantly constant: $\nabla_i g_{jk} = 0$ if ∇ is the covariant derivative defined via the Christoffel symbols associated to the metric g_{jk}.

2. $\langle \mathbf{5} \rangle$ Show that the covariant derivative satisfies the Leibniz rule $\partial_i(\phi_j v^j) = (\nabla_i \phi_j) v^j + \phi_j \nabla_i v^j$ where ϕ and v are a 1-form and a vector field.

3. $\langle \mathbf{8} \rangle$ Find the curvature biquadratic form $g(R(\partial_x, \partial_y)\partial_y, \partial_x)$ in the x-y tangent plane for the Poincaré metric $g = (dx \otimes dx + dy \otimes dy)/y^2$ on the upper half plane $y > 0$.