Fluid Dynamics, Autumn 2024, CMI

Assignment 9

Due by the beginning of the class on Monday, Nov 4, 2024

helicity, buoyancy force

1. $\langle \mathbf{4} \rangle$ *Divergence of a cross product.* Establish the vector identity $\nabla \cdot (\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{b} \cdot (\nabla \times \boldsymbol{a}) - \boldsymbol{a} \cdot (\nabla \times \boldsymbol{b})$ for vector fields $\boldsymbol{a}$ and $\boldsymbol{b}$. It is used in deriving a local conservation law for helicity in barotropic flow.

2. $\langle \mathbf{2} + \mathbf{2} + \mathbf{5} \rangle$ Find the vorticity and helicity density $\boldsymbol{v} \cdot \boldsymbol{w}$ of (a) a planar velocity field of the form $\boldsymbol{v} = (u(x, y), v(x, y), 0)$ and (b) $\boldsymbol{v} = (z, x, y)$ in Cartesian coordinates. (c) For the field in (b) solve the equations for streamlines and express $(x(s), y(s), z(s))$ in terms of three constants of integration.

3. $\langle \mathbf{3} + \mathbf{4} \rangle$ Suppose a body having the shape of a cuboid of (horizontal) surface area A and height h is fully submerged in a fluid of constant density ρ at rest and subject to Earth's downward acceleration due to gravity $-g\hat{z}$ where $\hat{z}$ points upwards. (a) Find the force due to fluid pressure on the cuboid. (b) Explain with a diagram how you would evaluate this force if the body had a more general shape and occupied a volume V_b. Propose a formula for the force due to pressure in this case.