Fluid Dynamics, Autumn 2024, CMI
Assignment 6
Due by the beginning of the class on Monday, Oct 7, 2024
Vorticity, vortex force, streamlines, Bernoulli equation

1. $\langle \mathbf{1+2+4} \rangle$ Find a simple example of a vector field $\boldsymbol{v}(r) = (v^x, v^y, v^z)$ in 3d whose curl $\boldsymbol{w} = \nabla \times \boldsymbol{v}$ is not perpendicular to $\boldsymbol{v}$ at least somewhere. State this condition in terms of dot/cross products. Begin by considering a unidirectional vector field and then try a vector field that has no component in some direction (integral curves lie in parallel planes).

2. $\langle \mathbf{5} \rangle$ Find a simple example of a vector field $\boldsymbol{v}$ for which the vortex force per unit mass is not identically nonzero.

3. $\langle \mathbf{3+2} \rangle$ Consider an incompressible steady velocity vector field $\boldsymbol{v} = u(x,y)\hat{x} + v(x,y)\hat{y}$ on the x-y plane. It may be expressed in terms of a stream function $\boldsymbol{v} = \nabla \times (\psi \hat{z})$. (a) Explain the relation between integral curves of $\boldsymbol{v}$, level curves of the stream function ψ and streamlines. (b) Suppose further that $\boldsymbol{v}$ is irrotational and may be represented as a potential flow $\boldsymbol{v} = \nabla\phi$. Explain the relation between level curves of the stream function ψ and equipotential curves (level curves of ϕ).

4. $\langle \mathbf{4} \rangle$ Consider a rigid body immersed in an unsteady constant density potential flow $\boldsymbol{v}(\boldsymbol{r}, t) = \nabla\phi(\boldsymbol{r}, t)$ in the absence of any body force. Show that the force on the body due to fluid pressure is expressible as $\boldsymbol{F} = \rho \int_S (\frac{\partial \phi}{\partial t} + \frac{1}{2} v^2) \hat{n} \, dS$ where $\hat{n}$ is the outward pointing normal to the surface S of the body. Generalize this formula to include the effect of a conservative body force per unit mass $\boldsymbol{f}/\rho = -\nabla\varphi$.