

Continuum Mechanics, Spring 2018 CMI

Problem set 9

Due at the beginning of lecture on Monday Apr 16, 2018

Equations of elastodynamics and fluid dynamics

1. **(4)** Express the continuity equation for mass conservation in terms of the material derivative.
2. **(6)** A fluid flow is incompressible if the volume of each material element does not change with time. Find an alternate characterization of incompressibility in terms of the Eulerian velocity field \mathbf{v} . Explain your reasoning.
3. **(10)** Recall the equations of elastodynamics we derived for the Eulerian variables ρ and \mathbf{v} in the absence of external body forces. Now suppose the elastic stress T^{el} is that due to pressure p .
 - (a) **(4)** Obtain the resulting non-linear equation for the evolution of \mathbf{v} .
 - (b) **(6)** Now suppose the motion is incompressible. Given \mathbf{v} and ρ find a second-order (non-evolutionary) PDE that the pressure must satisfy.