

Continuum Mechanics, Spring 2018 CMI

Problem set 6

Due at the beginning of lecture on Monday Mar 19, 2018

Elastostatics

1. **⟨16⟩** Consider a homogeneous isotropic material with Young's modulus E and Poisson's ratio ν . It is subject to a horizontal (along x) normal tensile stress g everywhere.
 - (a) **⟨8⟩** Using the previously determined displacement field $\xi = (g/E)(x, -\nu y, -\nu z)$ and stress tensor $T_{ij} = -g \text{diag}(1, 0, 0)$ in the tensorial form of Hooke's law, express E and ν in terms of Lamé's elastic constants μ and λ .
 - (b) **⟨5⟩** Invert the formulae obtained to express μ and λ in terms of E and ν . Show that λ and μ must be non-negative.
 - (c) **⟨3⟩** Express the bulk modulus K in terms of λ and μ and verify that the shear modulus defined earlier coincides with Lamé's elastic constant μ .
2. **⟨5⟩** Show that the displacement field in a homogeneous isotropic elastic material in equilibrium is bi-harmonic in the absence of external forces: $\nabla^2 \nabla^2 \xi = 0$.