Continuum Mechanics, Spring 2018 CMI

Problem set 2

Due at the beginning of lecture on Monday Jan 29, 2018

Wave equation, Noether's theorem and factorization of 2D Laplacian

1. $\langle \mathbf{12} \rangle$ A version of Noether's theorem states that if $u \to u + \delta u$ is an infinitesimal symmetry of the Lagrangian, then the Noether charge $Q = \int \pi(x,t)\delta u(x,t) dx$ is conserved. Here π is the momentum conjugate to u and $L = \int (\frac{1}{2}\rho u_t^2 - \frac{1}{2}\tau u_x^2) dx$ is the Lagrangian for small transverse oscillations of a stretched string, say on $-\infty < x < \infty$ with decaying BCs.

   (a) $\langle \mathbf{5} \rangle$ Consider small constant shifts $u \to u + \epsilon$. Show that L is invariant under such a shift and obtain the corresponding conserved quantity and interpret it.

   (b) $\langle \mathbf{7} \rangle$ Consider small constant shifts $x \to x + \epsilon$. Find δu. Show that L is invariant under this infinitesimal transformation and obtain the corresponding Noether charge and verify that it is conserved.

2. $\langle \mathbf{9} \rangle$ Suppose $z = x + iy$ and $\bar{z} = x - iy$ where x and y are real. Suppose f is a function of x and y or equivalently z and $\bar{z}$.

   (a) $\langle \mathbf{5} \rangle$ Express $\partial_z = \frac{\partial}{\partial z}$ and $\partial_{\bar{z}} = \frac{\partial}{\partial \bar{z}}$ as linear combinations of ∂_x and ∂_y when acting on f.

   (b) $\langle \mathbf{4} \rangle$ Factorize the Laplace operator in two dimensions $\partial_x^2 + \partial_y^2$ just as we did for d'Alembert's wave operator.