Classical Mechanics 2, Spring 2023 CMI
Assignment 1
Due by 6pm, Saturday Jan 7, 2023
Euler-Lagrange equations

1. $\langle \mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{3} \rangle$ Suppose a nonrelativistic particle of mass m moves on the real line with coordinate x and velocity $\dot{x}$ in an inertial frame. Find the Euler-Lagrange equation for the following Lagrange functions. Viewing the EL equation as the equation of motion, mention the force (give a formula and a line of explanation) the particle is subject to in each case. (a) $L = \frac{1}{2}m\dot{x}^2$, (b) $L = \frac{1}{2}m\dot{x}^2 + \alpha x\dot{x}$, (c) $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ and (d) $L = \frac{1}{2}m\dot{x}^2 - V(x)$ for some smooth function V and positive constants k, α. (e) Compare examples (a) and (b) and comment on any inferences you can draw.

2. $\langle \mathbf{1}+\mathbf{3}+\mathbf{3}+\mathbf{3} \rangle$ Suppose a particle of mass m moves on the Euclidean plane (with Cartesian coordinates x, y) subject to the potential $V(x, y)$. (a) Write down Newton's equations of motion for this particle as 2nd order ODEs. (b) Propose a Lagrangian $L(x, y, \dot{x}, \dot{y})$ such that its Euler-Lagrange equations reproduce Newton's equations of motion for this particle (show that this is the case). (c) Express the proposed L in polar coordinates and velocities, i.e., write it as a function $\tilde{L}$ of $r, \phi, \dot{r}, \dot{\phi}$. (d) Derive the EL equations for $\tilde{L}$ and write them as 2nd order ODEs for r, ϕ. Hint: You may use the formulae for the radial and angular velocities $\dot{r}$ and $\dot{\phi}$ derived earlier in terms of Cartesian variables and vice versa.