Classical Mechanics 2, Spring 2016 CMI
Problem set 9
Due by the beginning of lecture on Monday Mar 21, 2016
Inertia tensor of a rigid body

1. $\langle \mathbf{16} \rangle$ Consider a uniform square plate of side L and total mass M (surface mass density $\sigma = M/L^2$).

    (a) $\langle \mathbf{3} \rangle$ Select a convenient right-handed orthonormal coordinate system and draw a diagram of the plate and the coordinate axes.

    (b) $\langle \mathbf{1} \rangle$ Where is the center of mass located? What are its coordinates in the chosen system of coordinates?

    (c) $\langle \mathbf{8} \rangle$ Find all the matrix elements of the inertia matrix in a suitable basis with origin at the center of mass. Exploit the symmetries of the mass distribution on the plate to simplify the calculation.

    (d) $\langle \mathbf{2} \rangle$ Find the principal moments of inertia of the plate.

    (e) $\langle \mathbf{2} \rangle$ Find the corresponding principal axes of inertia, are they uniquely determined? Clarify.

2. $\langle \mathbf{5} \rangle$ Consider a rigid rotator, a system of mass points m_a, $a = 1, \cdots N$ that are concentrated along the z-axis at the locations z_a, defined so that the center of mass lies at $z = 0$ (and $x = y = 0$). Draw a figure indicating masses, origin and coordinate axes. What can you say about the off-diagonal elements of the inertia tensor? Find the principal moments of inertia of the rotator.