Classical Mechanics 2, Spring 2016 CMI
Problem set 8
Due by the beginning of lecture on Monday Mar 14, 2016
Generators for finite canonical transformations

1. $\langle \mathbf{15} \rangle$ Consider the finite canonical transformation, corresponding to a rotation of the phase plane

$$Q = cq - sp \quad \text{and} \quad P = sq + cp \quad \text{where} \quad s = \sin\theta \quad \text{and} \quad c = \cos\theta. \tag{1}$$

   (a) $\langle \mathbf{2} \rangle$ We seek a generating function of type-II $W(q,P)$ for the above finite CT. Write the equations of transformation for such a generator and find the two first order partial differential equations that $W(q,P)$ must satisfy to ensure it generates the above CT.

   (b) $\langle \mathbf{3} \rangle$ Integrate the PDE that involves only one partial derivative $\frac{\partial W}{\partial q}$ and express the answer in terms of an arbitrary differentiable function g. What variables does g depend on?

   (c) $\langle \mathbf{4} \rangle$ Determine g by imposing the other PDE. Find an ODE for g, solve it, and give a simple formula for the generating function $W(q,P)$.

   (d) $\langle \mathbf{1} \rangle$ For $\theta = 0$ what does $W(q,P)$ reduce to, and what CT does it generate?

   (e) $\langle \mathbf{1} \rangle$ Verify that your proposed function $W(q,P)$ indeed generates the above finite rotation.

   (f) $\langle \mathbf{2} \rangle$ Find a generating function of type $F_1(q,Q)$ that generates the same finite rotation via an appropriate Legendre transform from $W(q,P)$. This provides an example of a CT that admits a generator of both type I and II.

   (g) $\langle \mathbf{2} \rangle$ Try to find a generator of type I for the identity CT, by letting the angle of rotation go to zero. What do you find?