Classical Mechanics 2, Spring 2016 CMI

Problem set 6

Due by the beginning of lecture on Monday Feb 15, 2016

Canonical Transformations

1. $\langle \mathbf{17} \rangle$ Consider a free particle moving on the half line $q > 0$ with Lagrangian $L(q, \dot{q}) = \frac{1}{2} m \dot{q}^2$ and equation of motion $\ddot{q} = 0$. Suppose we make the change of coordinate to $Q = q^2$.

   (a) $\langle \mathbf{2} \rangle$ Express the equation of motion $\ddot{q} = 0$ as a second order differential equation for Q.

   (b) $\langle \mathbf{2} \rangle$ Find the new Lagrangian $\tilde{L}(Q, \dot{Q})$.

   (c) $\langle \mathbf{2} \rangle$ Find the momentum P conjugate to Q from the transformed Lagrangian. Express P as a function of Q and $\dot{Q}$ and as a function of q and p.

   (d) $\langle \mathbf{2} \rangle$ Find the new Hamiltonian $\tilde{H}(Q, P)$.

   (e) $\langle \mathbf{2} \rangle$ Find Hamilton's equations that follow from the new Hamiltonian $\tilde{H}(Q, P)$ [written as first order differential equations for Q and P].

   (f) $\langle \mathbf{2} \rangle$ Check that Hamilton's equations for Q, P are equivalent to the 2nd order ODE for Q obtained by transforming $\ddot{q} = 0$ above.

   (g) $\langle \mathbf{3} \rangle$ Calculate the Legendre transform of the new Lagrangian $\tilde{L}(Q, \dot{Q})$ and check that you get the new Hamiltonian $\tilde{H}(Q, P)$.

   (h) $\langle \mathbf{2} \rangle$ Find the Poisson bracket $\{Q, P\}$ (using definition of PB by differentiating in q and p) and compare with the canonical $\{q, p\}$ PB.