Classical Mechanics 2, Spring 2016 CMI
Problem set 12
Due by the beginning of lecture on Monday April 11, 2016
Rigid body, Euler angles

1. ⟨**12**⟩ Consider force-free motion of a symmetrical top with principal moments of inertia $I_1 = I_2 > I_3 \geq 0$. Let the Euler angle θ be the angle between the fixed angular momentum vector in space $\mathbf{L} = L\hat{Z}$ and the axis of the top along $\hat{x}_3 = \hat{z}$. We use the same conventions as in the lecture. Recall the solution of Euler's equations:

$$L_3 = \text{const}, \quad L_1 = C\cos(\omega t + \delta), \quad L_2 = C\sin(\omega t + \delta) \quad \text{where} \quad \omega = L_3\left(\frac{1}{I_1} - \frac{1}{I_3}\right). \tag{1}$$

   (a) ⟨**3**⟩ Relate ω to the generalized velocity $\dot{\psi}$ where ψ is the Euler angle defined earlier [Refer to our discussion on special choice of Euler angles for a symmetric top].

   (b) ⟨**2**⟩ We now consider the limiting case of a rigid rotator by letting $I_3 \to 0$ holding $I_1 = I_2$ and L fixed while ω remains finite. How must θ behave in the limit?

   (c) ⟨**4**⟩ From your knowledge of the motion of a rigid rotator and above formulae, what are the limiting values of $\frac{\cos\theta}{I_3}$, Ω_3 and ω?

   (d) ⟨**3**⟩ Qualitatively describe what happens to the manner of rotation of the symmetric top when it becomes a rigid rotator.