Classical Mechanics 2, Spring 2014 CMI

Problem set 5

Due by the beginning of lecture on Wednesday Jan 29, 2014

A pitfall in obtaining the Hamiltonian from Lagrangian

1. Suppose the kinetic energy and Lagrangian of a system is given by $L(q, \dot{q}) = T = \frac{1}{4}\dot{q}^4 - \frac{1}{2}\dot{q}^2$.

   (a) $\langle \mathbf{1} \rangle$ Plot T as a function of velocity.

   (b) $\langle \mathbf{1} \rangle$ Find the momentum p conjugate to q.

   (c) $\langle \mathbf{1} \rangle$ Plot p as a function of $\dot{q}$.

   (d) $\langle \mathbf{3} \rangle$ To go from Lagrangian to Hamiltonian we need to express the velocity in terms of momentum. Re-draw the previous graph of p versus $\dot{q}$. Argue from the graph and indicate on the graph whether and where one may solve for the velocity in terms of a given momentum uniquely/non-uniquely. Explicit formula for $\dot{q}$ as a function of p is not required.

   (e) $\langle \mathbf{3} \rangle$ Does $H = p\dot{q} - L$ with $p = \frac{\partial L}{\partial \dot{q}}$ define the hamiltonian as a single-valued function of position and momentum on phase space? Where do we run into trouble?