Classical Mechanics 2, Spring 2014 CMI

Problem set 11

Due by the beginning of lecture on Monday Mar 24, 2014

Inertia tensor.

1. $\langle \mathbf{18} \rangle$ Consider a uniform square plate of side L and total mass M.

   (a) $\langle \mathbf{3} \rangle$ Select a convenient right-handed orthonormal coordinate system and draw a diagram of the plate and the coordinate axes.

   (b) $\langle \mathbf{1} \rangle$ Where is the center of mass located? What are its coordinates in the chosen system of coordinates?

   (c) $\langle \mathbf{8} \rangle$ Find all the matrix elements of the inertia matrix in a suitable basis with origin at the center of mass. Exploit the symmetries of the mass distribution on the plate to simplify the calculation.

   (d) $\langle \mathbf{3} \rangle$ Find the principal moments of inertia of the plate and check whether they satisfy/saturate the triangle inequalities.

   (e) $\langle \mathbf{2} \rangle$ Find the corresponding principal axes of inertia, are they uniquely determined? Clarify.

   (f) $\langle \mathbf{1} \rangle$ Is the square plate a symmetric top? Why?

2. $\langle \mathbf{13} \rangle$ Now consider a uniform solid cube of side L and mass M.

   (a) $\langle \mathbf{3} \rangle$ Choose a convenient coordinate system with origin at the center of mass and draw a figure of the cube and coordinate axes.

   (b) $\langle \mathbf{8} \rangle$ Find the matrix elements of the inertia matrix in a suitable basis.

   (c) $\langle \mathbf{2} \rangle$ Find the principal moments of inertia of the cube and establish whether it is a spherical top.