Classical Mechanics 1, Autumn 2025 CMI
Assignment 6
Due by 6pm, 17 Oct, 2025
Damped motion, Spherical polars, Angular momentum

1. $\langle \mathbf{2+2+1+3} \rangle$ **Damped motion.** Suppose a particle of mass m moving on the real line with position $x(t)$ is subject to a conservative force arising from the potential $V(x)$ as well as a damping force $-\gamma \dot{x}$ with damping coefficient $\gamma > 0$. (a) Show that the 'energy' $E = \frac{1}{2} m \dot{x}^2 + V(x)$ is a nonincreasing function of time. (b) Suppose $V(x) = -\frac{1}{2} k x^2 + \frac{1}{3} g x^3$ with $k = 2, g = 3$. Draw a graph of $V(x)$. Qualitatively describe the nature of the motion for initial conditions (c) $x(0) = \dot{x}(0) = 0$ and (d) $x(0) = 1, \dot{x}(0) = 0$.

2. $\langle \mathbf{5} \rangle$ Consider the integral $\int_0^1 dx/x^a$. Show that the integrand $1/x^a$ has an **integrable divergence** (integrand diverges but integral is finite) at $x = 0$ for $0 < a < 1$ and a nonintegrable divergence for $a \geq 1$. What is the integral when it is finite?

3. $\langle \mathbf{5} \rangle$ Consider a particle moving in 3d Euclidean space. Its position vector is $\boldsymbol{r}$ relative to a fixed origin. Using a figure of **spherical polar coordinates** (r, θ, ϕ), express $\hat{r}$ as a linear combination of $\hat{x}, \hat{y}, \hat{z}$. Then express $\dot{\hat{r}}$ as a linear combination of

$$\begin{aligned} \hat{\theta} &= -\sin\theta \hat{z} + \cos\theta(\cos\phi \hat{x} + \sin\phi \hat{y}) \quad \text{and} \\ \hat{\phi} &= -\sin\phi \hat{x} + \cos\phi \hat{y}. \end{aligned} \tag{1}$$

4. $\langle \mathbf{5} \rangle$ Consider a particle moving in 3d Euclidean space. Its position vector is $\boldsymbol{r}$ relative to a fixed origin and its momentum is $\boldsymbol{p} = m\dot{\boldsymbol{r}}$ where m is its mass. Its **angular momentum** with respect to the chosen origin is defined as $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$. The Cartesian components of $\boldsymbol{L}$ are denoted L_x, L_y, L_z. In spherical polar coordinates, show that the z component of angular momentum L_z of the particle can be expressed as $L_z = m r^2 \sin^2\theta \, \dot{\phi}$.