Classical Mechanics 1, Autumn 2025 CMI
Assignment 5
Due by 6pm, Fri 19 Sep, 2025
Newton's laws, dimensional analysis

1. $\langle \mathbf{4} \rangle$ **Conservation of momentum.** Consider a system of two point particles A and B of masses m_a and m_b. A exerts a force $\boldsymbol{F}_b$ on B and B exerts a force $\boldsymbol{F}_a$ on A. Write Newton's 2^{nd} law equations of motion for this system in an inertial frame where the position vectors of the particles are $\boldsymbol{r}_a$ and $\boldsymbol{r}_b$. Use Newton's laws to show that the total momentum of the system is independent of time.

2. $\langle \mathbf{4} \rangle$ **Conserved energy.** Use a suitable 'integrating factor' to derive a conserved energy for Newton's equation $m\ddot{x}_i = f_i$ for a particle moving in 3d subject to a potential $V(x_1, x_2, x_3)$ so that it experiences a force with Cartesian components $f_i = -\frac{\partial V}{\partial x_i}$ for $i = 1, 2, 3$.

3. $\langle \mathbf{1+1+1+2} \rangle$ **Dimensional analysis.** Find the dimensions ($M^\alpha L^\beta T^\gamma$) of (a) Newton's gravitational constant G, (b) Planck's constant h from the formula $E = h\nu$ where E and ν are the energy and frequency of a photon, (c) GM_e/c^2 where M_e is the mass of the Earth and c the speed of light and (d) the probability amplitude $\psi(x)$ for finding an electron within dx of x, given that the total probability $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ for finding the electron somewhere on the real line is one.

4. $\langle \mathbf{4} \rangle$ **Secant hyperbolic-squared potential.** Find the force (vector) corresponding to the 1d potential $V(x) = V_o \operatorname{sech}^2(x/l)$ for a fixed length $l > 0$. For $V_o > 0$, explain whether it is a repulsive or attractive potential for particles coming in from $\pm\infty$. Plot $V(x)$.