Classical Mechanics 1, Autumn 2025 CMI

Assignment 3

Due by 6pm, Fri 29 Aug, 2025

trajectory, gradient

1. $\langle \mathbf{1+4} \rangle$ Suppose a particle has a trajectory in 3d space given by the position vector $\boldsymbol{r}(t) = \cos t\ \hat{x} + \sin t\ \hat{y} + t\ \hat{z}$. (a) Find the velocity and acceleration vectors. (b) Plot the trajectory and describe its shape in words. Indicate the directions of the velocity and acceleration vectors at a point on the trajectory and say in words which way they point.

2. $\langle \mathbf{4+5} \rangle$ **Gradient of a scalar field.** Consider the scalar field $\psi = 1/\sqrt{x^2+y^2+z^2}$ defined using Cartesian coordinates in $\mathbb{R}^3$ with the origin left out. (a) Find the gradient of ψ for x, y, z not all zero. After obtaining the answer in Cartesian coordinates, re-express ψ and the final answer for the gradient in spherical polar coordinates. (b) Say in words which way $\nabla\psi$ points and why and how its magnitude varies. Plot this vector field on a suitably chosen plane through the origin. Does the choice of plane matter? Why or why not?

3. $\langle \mathbf{2+2+4} \rangle$ **Force from a potential.** Suppose a potential in 3d Euclidean space is given by $V = \alpha/r^n$ where r is the radial distance from the origin, $\alpha > 0$ a real constant and n a real number. (a) Find the associated force $\boldsymbol{F} = -\nabla V$ and express it in spherical polar coordinates. (b) For $n = -2$ write V and $\boldsymbol{F}$ in Cartesian coordinates. (c) Describe the resulting $\boldsymbol{F}$ for $n = -2$ by saying whether the force is attractive/repulsive (relative to the origin), how it behaves with distance and give it a suitable name using standard terminology. Plot the graph of $V(r)$.