

Classical Mechanics 1, Autumn 2025 CMI

Assignment 1

Due by 6pm, Monday Aug 11, 2025

Vectors, dot and cross products

1. $\langle 4 \rangle$ How does the torque $\boldsymbol{\tau}$ due to a force \boldsymbol{F} on a particle change if the origin \boldsymbol{O} for position vectors is shifted to $\boldsymbol{O}' = \boldsymbol{O} + \boldsymbol{a}$? Relate the torques with respect to the two origins. Draw a figure to illustrate.
2. $\langle 2 + 3 \rangle$ A particle of mass m moves along a circle (say, uniformly counterclockwise) on a horizontal plane. Its angular momentum vector is defined as $\boldsymbol{L} = m\boldsymbol{r} \times \boldsymbol{v}$ where \boldsymbol{r} is its position vector and \boldsymbol{v} its instantaneous velocity. Comment on the direction of \boldsymbol{L} (upwards, downwards) and illustrate with figures in each of the following cases: (a) the origin is at a point inside the disk defined by the circle and (b) the origin is at a point on the horizontal plane lying outside the disk defined by the circle.
3. $\langle 3 \rangle$ Suppose $\boldsymbol{\Omega}$ and \boldsymbol{r} are a pair of vectors. Find an expression for $|\boldsymbol{\Omega} \times \boldsymbol{r}|^2$ purely in terms of dot products of the vectors $\boldsymbol{\Omega}$ and \boldsymbol{r} .
4. $\langle 3 \rangle$ Suppose a particle is subject to a force of the form $\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{h}$ where \boldsymbol{v} is its velocity and \boldsymbol{h} some vector. Comment on the work done by this force in displacing the particle infinitesimally.
5. $\langle 5 \rangle$ Consider a particle that moves on a sphere, its position vector \boldsymbol{r} is defined relative to the center of the sphere. The sphere rotates (counterclockwise when viewed from above) about a vertical axis through its center at the angular speed Ω . The angular velocity vector $\boldsymbol{\Omega}$ has magnitude Ω and points upwards. Explain the direction of the vector $-\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$. Draw a figure to clarify.