Classical Mechanics 1, Autumn 2022 CMI
Problem set 8
Due by 6pm, Friday Nov 4, 2022
Simple plane pendulum

1. ⟨**10**⟩ Linearize the equation of motion $\ddot{\theta} = -\omega^2 \sin\theta$ for a pendulum around $\theta = \pi$ (top of circle), i.e., put $\theta = \pi + \phi$ and suppose ϕ is treated to linear order. Here $\omega = \sqrt{g/\ell}$.

   (a) ⟨**2**⟩ Find the linear differential equation satisfied by ϕ.

   (b) ⟨**4**⟩ Use a suitable guess to find two linearly independent solutions of the equation for ϕ and the general solution of the equation for given $\phi(0)$ and $\dot{\phi}(0)$.

   (c) ⟨**4**⟩ Physically interpret the consequences of the solution of the linear differential equation for ϕ for typical (nonexceptional) initial conditions and comment on what it means for the stability of the static solution $\theta = \pi$. Comment on any exceptional ICs and the resulting time evolution.

2. ⟨**9**⟩ Consider a simple pendulum with bob of mass m and rod of length l subject to Earth's acceleration due to gravity with magnitude g. Denote the counterclockwise deflection angle θ and use the coordinate frame developed in the lecture.

   (a) ⟨**3**⟩ Find a expression (in terms of θ) for the torque $\boldsymbol{\tau}$ on the bob about the pivot. Which forces do/do not contribute to $\boldsymbol{\tau}$?

   (b) ⟨**1**⟩ Write an expression (in terms of θ) for the angular momentum $\boldsymbol{L}$ of the bob about the pivot.

   (c) ⟨**2**⟩ Use the evolution equation for $\boldsymbol{L}$ to derive a differential equation for $\theta(t)$. How does it compare with the EOM for θ derived from Newton's second law?

   (d) ⟨**3**⟩ Propose a limit holding m, l, g fixed in which the angular momentum is asymptotically conserved. Specify the limit through a suitable comparison of physical quantities. Comment on the nature of motion in this limit.