Classical Mechanics 1, Autumn 2022 CMI
Problem set 7
Due by 6pm, Wednesday Dec 1, 2022
Collisions, Time period of 1d motion, stability of equilibria

1. $\langle \mathbf{13} \rangle$ Consider one-dimensional elastic scattering of 2 particles of masses m_1, m_2 that retain their identities and masses after the collision. We view the collision in the lab frame. Suppose the velocities before and after collision are v_1, v_2 and v_1', v_2'. (a) $\langle \mathbf{1} \rangle$ Write the conservation equations for linear momentum and energy. You may use the symbols p for the conserved momentum, T for energy and $M = m_1 + m_2$ for the total mass. (b) $\langle \mathbf{1+2} \rangle$ Eliminate $v_1' = (p - m_2 v_2')/m_1$ and show that

$$v_2' = M^{-1}\left[p \pm \sqrt{p^2 - M(m_1/m_2)(p^2/m_1 - 2T)}\right] \tag{1}$$

(c) $\langle \mathbf{3} \rangle$ Argue from this that $v_2' = v_2$ or

$$v_2' = (2m_1 v_1 + (m_2 - m_1)v_2)/M. \tag{2}$$

(d) $\langle \mathbf{2+2} \rangle$ In the case of nontrivial scattering show that

$$v_1' = [2m_2 v_2 + (m_1 - m_2)v_1]/M. \tag{3}$$

Comment on the relation between the formulae for v_1' and v_2' and why the relation is to be expected. (e) $\langle \mathbf{2} \rangle$ Suppose the particles had equal and opposite initial velocities (i.e., as $t \to -\infty$). Under what conditions would the more massive particle come to rest after the collision (i.e., as $t \to \infty$)? Under these conditions, what is the velocity of the lighter particle after the collision?

2. $\langle \mathbf{6} \rangle$ A point particle of mass $m > 0$ moves along the real line subject to a force that arises from the potential $V(x) = \lambda(x^2 - a^2)^2$ where $\lambda, a > 0$ are fixed constants. (a) Plot the potential. (b) Find all static solutions of the equation of motion. (c) Classify them as stable and unstable to small perturbations.

3. $\langle \mathbf{5} \rangle$ Consider one-dimensional motion of a particle of mass m in a potential $V(x)$ with $x \in \mathbb{R}$. Suppose the particle's energy is E and $x_1 < x_2$ are a pair of neighboring turning points for oscillatory motion [$V(x) < E$ for $x_1 < x < x_2$]. Find an integral expression for the time period of the oscillatory motion between x_1 and x_2.