Classical Mechanics 1, Autumn 2022 CMI

Problem set 1

Due by 6pm, Monday Aug 8, 2022

Vectors, dot and cross products

1. $\langle \mathbf{4} \rangle$ How does the torque $\boldsymbol{\tau}$ due to a force $\boldsymbol{F}$ on a particle change if the origin $\boldsymbol{O}$ for position vectors is shifted to $\boldsymbol{O'} = \boldsymbol{O} + \boldsymbol{a}$? Relate the torques with respect to the two origins. Draw a figure to illustrate.

2. $\langle \mathbf{6} \rangle$ A particle of mass m moves along a circle on a horizontal plane. Its angular momentum vector is defined as $\boldsymbol{L} = m\boldsymbol{r} \times \boldsymbol{v}$ where $\boldsymbol{r}$ is its position vector and $\boldsymbol{v}$ its instantaneous velocity. Comment on the direction of $\boldsymbol{L}$ (upwards, downwards) and illustrate with figures in each of these cases (a) the origin is at the center of the circle and (b) the origin is at a point on the horizontal plane lying outside the disk defined by the circle.

3. $\langle \mathbf{3} \rangle$ Suppose $\boldsymbol{\Omega}$ and $\boldsymbol{r}$ are a pair of vectors. Find an expression for $|\boldsymbol{\Omega} \times \boldsymbol{r}|^2$ purely in terms of dot products of the vectors $\boldsymbol{\Omega}$ and $\boldsymbol{r}$.

4. $\langle \mathbf{3} \rangle$ Suppose a particle is subject to a force of the form $\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{h}$ where $\boldsymbol{v}$ is its velocity and $\boldsymbol{h}$ some vector. Comment on the work done by this force in displacing the particle infinitesimally.

5. $\langle \mathbf{5} \rangle$ Consider a particle that moves on a sphere, its position vector $\boldsymbol{r}$ is defined relative to the center of the sphere. The sphere rotates (counterclockwise when viewed from above) about a vertical axis through its center at the angular speed Ω. The angular velocity vector $\boldsymbol{\Omega}$ has magnitude Ω and points upwards. Comment on the direction of the vector $-\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$. Draw a figure to clarify.