Classical Mechanics (PG), Autumn 2013 CMI
Problem set 6
Due at the beginning of lecture on Wednesday August 28, 2013
Phase portrait

1. $\langle 16 \rangle$ Consider a particle in the potential $V(x) = g(x^2 - a^2)^2$ in one dimension. Here $g, a > 0$.

   (a) $\langle 2 \rangle$ Write a formula for the conserved energy. Find all equilibrium points (mention stable and unstable) and their energies.

   (b) $\langle 2 \rangle$ Roughly sketch the potential as a function of x, indicating the point a and the value of energy at the unstable equilibrium point.

   (c) $\langle 1 \rangle$ Give a suitable name for this potential.

   (d) $\langle 11 \rangle$ Draw a phase portrait for this system, i.e., indicate the phase space trajectories for various different values of energy/initial conditions. Do this without solving the equations of motion, but using conservation of energy, previous results and physical reasoning. You must draw at least 8 qualitatively distinct trajectories (with arrow showing direction of motion) and briefly mention the nature of the motion along each of them.