Parton distributions from $1+1$ QCD

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Parton Distributions from 1+1 QCD

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Abstract. We study the large N_c limit of a previously introduced reformulation of 2d QCD, HadronDynamics. This model is used for an effective description of the baryon in Deep Inelastic Scattering, when transverse momenta of partons are ignored. This allows us to determine the non-perturbative initial condition for Q^2 evolution equations: x_B dependence of structure functions at an initial Q_0^2. After including effects of transverse momenta via the DGLAP evolution, we compare our prediction with experimental measurements of xF_3 and find good agreement. We have only two parameters: the initial scale Q_0 and the fraction of baryon momentum carried by valence quarks.

The Q^2 evolution of the hadronic structure functions of Deep Inelastic Scattering (DIS) can be determined within the framework of perturbative QCD [1,4]. However, the initial condition for the DGLAP equations, the dependence on x_B at an initial Q_0^2 is non-perturbative and we do not yet have a reliable way of calculating it from QCD. Therefore, we have had to rely on parametrizations of experimental data for these initial conditions [4]. Here we study Rajeev's bi-local reformulation of 2 dimensional QCD, HadronDynamics [2]. This model is proposed as an effective description of the baryon at a low value of Q^2, where the transverse momenta of partons are ignored. The solutions of this model are used as initial conditions for DGLAP evolution (which can be thought of as including corrections due to transverse momenta [1]). The iso-spin averaged valence quark distribution determined this way compares well with experimental measurements of the structure function $xF_3(x_B, Q^2)$. Our only parameters are Q_0^2 and the fraction of baryon momentum carried by valence quarks, f.

We take as our effective hamiltonian for the leading 'relevant' interactions of the quarks, the hamiltonian of 2d Hadron Dynamics [2]. This is the same as the hamiltonian of 2d QCD in the null gauge $A_- = 0$, expressed in terms of the gauge invariant meson field $\hat{M}^a_b(x,y) = \frac{2}{N_c} : \chi_{b\alpha}(x)\chi^{\dagger a\alpha}(y) :$, where x and y lie along a null line. The longitudinal gluons induce a linear potential between quarks. In the large N_c limit, the ground state of the baryon (Baryon Number $= -\frac{1}{2}trM = 1$) is

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determined by minimizing the energy

$$\frac{E[M]}{N_c} = -\frac{1}{4}\int [p + \frac{{\mu_a}^2}{p}]\tilde{M}^a_a(p,p)\frac{dp}{2\pi} + \frac{\tilde{g}^2}{8}\int M^a_b(x,y)M^b_a(y,x)|x-y|dxdy. \quad (1)$$

μ_a^2 is related to the current quark masses m_a by a finite renormalization: $\mu_a^2 = m_a^2 - \frac{\tilde{g}^2}{\pi}$ [5] and $\tilde{g}^2 = g^2 N_c$. Though the hamiltonian is quadratic, this is an interacting theory since the phase space is a curved manifold due to the quadratic constraint arising from the Pauli principle for quarks, $(\epsilon + M)^2 = 1$ (ϵ is the hilbert transform).

The diagonal entries of the (normal ordered) density matrix, $-\frac{1}{2}\tilde{M}^a_a(p,p)$ and $-\frac{1}{2}\tilde{M}^a_a(-p,-p), 0 \le p \le P$ are the quark and anti-quark probability densities. Experimentally, it is inferred that gluons carry about half the baryon momentum at low Q_0^2 [4]. Since we have ignored the transversely polarized gluons, we require that the valence quarks $-\frac{1}{2}\sum_{a=u,d}(M^a_a(p,p) - M^{\bar{a}}_{\bar{a}}(-p,-p))$, $p > 0$ carry only a fraction f of the total baryon momentum P.

We have found the *exact* minimum of this variational principle in the chiral and large N_c limit [3]. The minimum occurs for a configuration consisting of valence quarks alone: $\tilde{M}_1(p,q) = -2\tilde{\psi}(p)\tilde{\psi}^*(q)$ for $p,q \ge 0$ and zero otherwise. Indeed, $\tilde{\psi}(p)$ can be regarded as the valence quark wave function, and in the ground state $\tilde{\psi}(p) = \sqrt{\frac{2\pi}{f\bar{P}}}\exp\left(\frac{-p}{2f\bar{P}}\right)$, where $\bar{P}$ is the mean baryon momentum per color. For finite N_c, we find that the functional form $\tilde{\psi}(p) = Cp^a(1-\frac{p}{P})^b, 0 \le p \le P$ provides a good approximation to the ground state with $b = \frac{N_c}{2f} - 1 + a(\frac{N_c}{f} - 1)$ and $a \to \sqrt{\frac{3}{\pi}}\frac{m}{\tilde{g}}$ for small current quark masses.

The above purely valence quark configuration is the exact minimum in the chiral and large N_c limits. To estimate the anti-quark and sea quark content of the baryon away from chiral symmetry, we must allow for more general configurations. The simplest baryon number one configuration that departs from the valence quark approximation and satisfies the quadratic constraints can be expressed as:

$$M_3 = -2\psi\otimes\psi^\dagger + 2\zeta_-^2[\psi_-\otimes\psi_-^\dagger - \psi_+\otimes\psi_+^\dagger] + 2\zeta_-\zeta_+[\psi_-\otimes\psi_+^\dagger + \psi_+\otimes\psi_-^\dagger]. \quad (2)$$

Here ψ, ψ_+ are orthonormal and non-vanishing only for positive momenta; they are the valence and sea quark wave functions. ψ_- vanishes for positive momenta and describes anti-quarks. $0 \le \zeta_- \le 1$ measures the deviation from the purely valence quark configuration. $\zeta_+ = \sqrt{1-\zeta_-^2}$. Baryon number is given by $B = \int_0^\infty \left\{|\tilde{\psi}(p)|^2 + \zeta_-^2\left[|\tilde{\psi}_+(p)|^2 - |\tilde{\psi}_-(-p)|^2\right]\right\}\frac{dp}{2\pi}$. From our previous result we expect ζ_- to vanish as $\frac{m^2}{\tilde{g}^2} \to 0$. For physically relevant current quark masses ($\frac{m^2}{\tilde{g}^2} \sim 10^{-3}$), our estimates [3] show that the corrections due to anti and sea-quarks is negligible; for instance they carry less than a percent of baryon momentum.

Thus, in the chiral limit, our approximation for the valence quark probability distribution is $V(x_B, Q_0^2) = C(1-x_B)^{\frac{N_c}{f}-2}$ (For $f = 0$ this agrees with numerical computations of Hornbostel et. al. [6]). We can compare this prediction

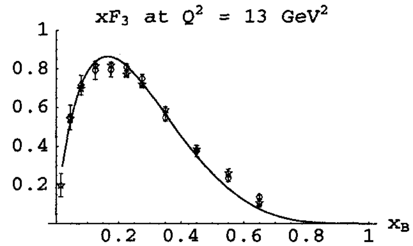

FIGURE 1. Predicted xF_3 compared with measurements by CDHS($\diamondsuit$) and CCFR($\star$).

with experimental data after including corrections due to transverse momenta by evolving the distribution to higher Q^2 via the DGLAP equation. The valence quark parton distribution function is given by $\phi^V(x_B, Q_0^2) = \nu(Q_0^2)V(x_B, Q_0^2)$, where $\nu(Q_0^2) = \int_0^1 dx_B \phi^V(x_B, Q_0^2)$ is determined by solving the integrated DGLAP equation with initial condition $\nu(\infty) = N_c = 3$ from $Q^2 = \infty$ to Q_0^2. Within the leading logarithmic approximation, $\phi^V(x_B, Q^2)$ is the same as $F_3(x_B, Q^2)$ averaged over neutrino and anti-neutrino scattering on an isoscalar target. Then $\nu(Q_0^2) = \int_0^1 dx_B F_3(x_B, Q_0^2)$, is given by the GLS sum rule [7]. We compare our prediction for xF_3 with measurements by the CDHS and CCFR collaborations in Fig. 1 and find good agreement for a choice of parameters $Q_0^2 \sim 0.4 GeV^2$ and $f \sim \frac{1}{2}$. Thus HadronDynamics is a successful way of deriving structure functions from collinear QCD.

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