SAMPLE QUESTIONS

(1) Show that a product of two manifolds without boundary is a manifold. What can you say when one of them have boundary? What can you say when both of them have boundary.
(2) Let $B(0,1)$ denote the open unit ball in $\mathbb{R}^n$. Show that the map $f : B(0,1) \to \mathbb{R}^n$ given by

$$f(x) = \frac{x}{1-|x|^2}$$

is a diffeomorphism of the ball with $\mathbb{R}^n$
(3) Show that there is no metric on S^2 having curvature bounded above by 0 and no metric on surface of genus g which is bounded below by 0.
(4) Let U be an open subset of a manifold X. Show that for any $x \in U$ the tangent space at x to U, i.e., $T_x(U)$ is same as T_xX.
(5) Let f from $\mathbb{R} \to \mathbb{R}$ be a local diffeomorphism (diffeomrophism in a neighborhood of each point). Show that image of $\mathbb{R}$ under f is an open interval. Moreover, f is a diffeomorphism of $\mathbb{R}$ on to $f(\mathbb{R})$.
(6) Prove that paraboloid $x^2 + y^2 - z^2 = a$ define a manifold when $a > 0$. Why is it not a manifold when $a = 0$.