Chennai Mathematical Institute

Topology : Test 1
Instructor: Prof. P. Vanchinathan

February 5, 2009

Answer all questions for a maximum of 40 marks

1. Let A, B be subsets of a topological space X. Then show that $\overline{A} \cup \overline{B} = \overline{A \cup B}$.

2.For a subspace $A \subset X$, a point $x \in X$ is said to be in the boundary of A if it is a limit point for both A and its complement in X. Show that U is open iff $\overline{U} - U$ is the boundary of U.

3.Let f, g be two real-valued functions defined on a topological space X that are continuous there (for the usual topology on the real numbers). Then show that the function h defined by $h(x) = \min\ \{f(x), g(x')\}$

4.Let $f : X \to Y$ and $g : X' \to Y'$ be continuous maps. Show that the function $f \times g : X \times X' \to Y \times Y'$ defined by $(f \times g)(x, x') = (f(x), g(x'))$ is continuous for the product topologies.

5.Let $\{U_\alpha\}_\alpha$ be a collection of open sets in a topological space X such that their union is the whole space X. A subset $Y \subset X$ has the property that $Y \cap U_\alpha$ is closed in U_α for every α. Then, show that Y is a closed set in X.

6.Construct $F : \mathbf{R} \to \mathbf{R}^n$, ($n$ any positive integer) a continuous function for the usual topologies such that the image of F is not contained in any **proper** vector subspace of $\mathbf{R}^n$.

For any queries call me(Prof. Vanchinathan) at: 9940132501.