Quantum Mechanics I:
Quiz

Total: 15 marks

NOTE: The harmonic oscillator creation-annihilation operators are
$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}(x - \frac{ip}{m\omega})$ and its Hermitian conjugate: they satisfy $[a, a^\dagger] = 1$.
The action on the states is $a|n\rangle = \sqrt{n}|n-1\rangle, \; a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

(1) From the Heisenberg equation of motion, find the time evolution equation for the operator $V(x) = \frac{1}{2}m\omega^2 x^2$ corresponding to the potential energy (using the expression for product operator commutators). [4 mks]

(2) Say the harmonic oscillator is in the ground state $|0\rangle$ initially. It is subjected to a sharp impulse-like interaction that switches on for a very short time Δt and then switches off. The Hamiltonian for this interaction is $H_{int} = \lambda x^4$, where x is the position coordinate.
(a) Expand out the time evolution operator $U_{int}(t) = e^{-iH_{int}t}$ to linear order for this impulse interaction. This is a good approximation since Δt is very small (and the interaction strength λ is also assumed to be small). [3 mks.]
(b) Evaluate carefully the time evolved state after the interaction has switched off using the evolution operator in (a), using the expression for the position operator in terms of the creation-annihilation operators. [5 mks]
(c) Calculate the probability that the system, after the impulse interaction has switched off, is in the excited state $|n\rangle$. For nonzero probability, how large can n be under the assumptions and approximations of (a), (b) ? What happens when the time Δt of interaction is increased ? [3 mks]