

# Mathematical Physics - I Aug-November 2008

## 1. Lecture 1

- (a) What is mathematical physics ?
- (b) Scalars and vectors.
- (c) Coordinate systems. Representations of scalars and vectors.
- (d) Addition of vectors. The parallelogram law.
- (e) Scalar multiplication and linear combinations.

## 2. Lecture 2

- (a) Scalar products of vectors. Basis vectors.
- (b) The vector product.
- (c) The scalar triple product

## 3. Lecture 3

- (a) Coordinate transformations. Rotations and scaling.
- (b) Geometric definition of vectors.

## 4. Lecture 4

- (a) Curves: A one parameter family of vectors.
- (b) Differentiation and integration of vectors. Tangents.
- (c) Surfaces: A two parameter family of vectors.
- (d) Tangents and normals.

## 5. Lecture 5

- (a) Scalar and vector fields.
- (b) Differentiation of fields.
- (c) The gradient and directional derivative.
- (d) The divergence of a vector field.
- (e) Lines of Force (integral curves).

## 6. Lecture 6

- (a) The curl of a vector field.
- (b) Divergenceless fields
- (c) Irrotational fields.

**7. Lecture 7**

- (a) Line integrals.
- (b) Surface integrals.
- (c) Volume integrals.

**8. Lecture 8**

- (a) Gauss theorem.

**9. Lecture 9**

- (a) Stokes Theorem.

**10. Lecture 10**

- (a) Curvilinear coordinates. Cylindrical and spherical coordinates.
- (b) Differentiation and integration of scalars. The jacobian.

**11. Lecture 11**

- (a) Vector components in cylindrical and spherical coordinates.
- (b) Gradient, divergence and curl.

**12. Lecture 12**

- (a) General complex linear vector space.
- (b) Linear operations and matrices.
- (c) Matrix multiplication.
- (d) Transposition and conjugation. Non-square matrices, row and column vectors.
- (e) Inner products and the triangle inequality.

**13. Lecture 13**

- (a) Linear equations.
- (b) Inverse and determinant.
- (c) Properties of the determinant.

**14. Lecture 14**

- (a) Eigenvalues and eigenvectors.
- (b) The secular determinant.
- (c) Hermitian, orthogonal and unitary matrices.

**15. Lecture 15**

- (a) Diagonalisation of Hermitian matrices.
- (b) Left and right eigenvectors of general matrices.

**16. Lecture 16**

- (a) Direct products of vector spaces.
- (b) Tensors.

**17. Lecture 17**

- (a) Vector spaces of functions.
- (b) Linear operators, Kernals.
- (c) Differential operators.
- (d) Eigenvalues and eigenfunctions of differential operators

**18. Lecture 18**

- (a) Fourier basis for  $N$  dimensional vector spaces.
- (b) Discrete Fourier transforms.
- (c) Fourier basis for spaces of functions.
- (d) Continous Fourier transforms.

**Lecture 19**

- (a) The uncertainty principle.

- (b) Convolutions.
- (c) Power spectrum.
- (d) Multiplying signals.
- (e) Filtering signals.

19. **Lecture 20**

- (a) First order ordinary differential equations (ODE).
- (b) Second order ODE's. Newtons law.
- (c) Reduction to first order. Phase space.

20. **Lecture 21**

- (a) Forced, damped harmonic oscillator, resonance.
- (b) Greens functions.

21. **Lecture 22**

- (a) General linear differential equations.
- (b) Bessels functions. Legendre polynomials.

22.

23. **Lecture 23**

- (a) The Poisson equation.

**Lecture 24**

- (a) The heat equation.

**Lecture 25**

- (a) The wave equation.