EXPERIMENT 2:Determination of Moment of Inertia(I_R) of a ring using torsional pendulum

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1 Aim of Experiment

We will be measuring the Moment of Inertia (I_R) of a ring using the torsional properties of a wire and a body of known Moment of Inertia

2 Apparatus required

a)A stand with clamp
b)A straight wire
c)A disk of known weight and diameter
d)Stop watch
e)Screw gauge
f)Meter scale

3 Theory of experiment

Brief discussion and relevant formulae:

A torsion pendulum or torsional oscillator consista of a disk-like mass suspended from thin rod. When the mass is twisted about the axis of the wire , the wire exerts a torque on the mass, tending to rotate it back to its original position. If twisted and released, the mass will oscillate back and forth to its original position executing a simple harmonic motion. The restoring torque is proportional to the angular displacement.

$$\tau = -C\theta \tag{1}$$

The time period (T) of torsional oscillation of mass and wire system is given by:

$$T = 2\pi\sqrt{\frac{I}{C}} \tag{2}$$

where I =Moment of Inertia of the disk
C =Couple per unit angle of the twist

$$C = \frac{\eta \pi r^4}{2l} \tag{3}$$

where r =Radius of wire
l =Length of suspension wire
η =Rigidity modulus of material of wire
The equation of motion for torsional pendulum

$$I\frac{d^2\theta}{dt^2} = -C\theta \tag{4}$$

$$\frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta \tag{5}$$

The solution to this differential equation is given by:

$$\theta_t = \theta_m(cos\ \omega t + \phi) \tag{6}$$

$$T = 2\pi\sqrt{\frac{I}{C}} \tag{7}$$

Experimental study

By providing angualr twist to the mass the time periods can be estimated:

- Time perion with disk alone (T_D) and disk with ring (T_{DR}) can be obtained.
- The Moment of Inertia of the disk can be obtained by measuring the mass and radius of the disk and by using the formula $I_D = \frac{1}{2}MR^2$ We know,

$$C = \frac{4\pi^2 I_D}{{T_D}^2} = \frac{4\pi^2(I_D + I_R)}{{T_{RD}}^2} \tag{8}$$

where I_D=Moment of Inertia of disk alone
I_R=Moment of Inertia of ring

$$\frac{{T_{RD}}^2}{{T_D}^2} = \frac{I_D + I_R}{I_D} = 1 + \frac{I_R}{I_D} \tag{9}$$

$$I_R = I_D\left(\frac{{T_{RD}}^2}{{T_D}^2} - 1\right) \tag{10}$$

Therefore knowing I_D which can be estimated by determining its mass and measuring the radius R

- The Moment of Inertia of the ring can be evaluated theoretically and a comparison with experimental value can be done. We can use the Moment of Inertia of ring+disk to estimate the rigidity modulus of the wire. Then, we can compare with the standard values to verify our results.
- The rigidity modulus of brass wire = $37.3 \times 10^9\ N/m^2$
- Time period may be estimated by timing oscillation at least for 20 oscillation.
- Mass can be determined using the digital balance.

4 Procedure

Firstly, we remove the wire of any kinks or undulations so as to guarantee a minimum systematic error. Then, the weight of the disk and radius of the disk is measured using a digital balance and meter scale respectively. From that, we can calculate the Moment of Inertia of the disk(I_D). The disk is now hung from hte wire which is clamped to the stand. The disk is given a small rotation in any sense i.e clockwise/anticlockwise. This causes a torsion in the brass wire and the disk begins to oscillate about a mean position. We measure the time of 5 oscillations with the help of a stopwatch. Measuring only one single oscillation would have been erraneous because of personal measurement errors. Now, we place the ring on top of the disk. Now, we measure the time period of the disk+ring system. Here again, we adopt the same method for measuring the time period in order to minimize the error. From the two time periods and knowing the Moment of Inertia of the disk, we can easily calculate the Moment of Inertia of the ring. This same process is done with different lengths. The mean gives us the value of the Moment of Inertia of the ring.

5 Calculations

Weight of ring = 327 gm
Weight of disc = 920 gm
Radius of wire = 0.68 cm(obtained from screw gauge)
The screw gauge had zero instrumental error i.e the circular scale zero coincided with the linear scale zero mark.
Radius of disk = 6cm
Inner radius of ring = 4.95cm

5.1 Table 1-Time period of oscillation of disk

S.No.	Length of wire *cm*	Time of 5 osc. *s*	Time Period *s*
1	30.3	26.03	5.21
2	30.3	26.09	5.22
3	30.3	26.14	5.23
4	41	30.06	6.01
5	41	30.15	6.03
6	41	30.20	6.04
7	49.2	32.66	6.53
8	49.2	32.59	6.52
9	49.2	32.72	6.54

5.2 Table 2-Time period of ring+disk

S.No.	Length of wire *cm*	Time of 5 osc. *s*	Time Period *s*
1	30.3	32.96	6.59
2	30.3	32.96	6.59
3	30.3	33.10	6.62
4	41	38.01	7.60
5	41	38.02	7.60
6	41	38.14	7.63
7	49.2	41.31	8.26
8	49.2	41.25	8.25
9	49.2	41.08	8.22

From the weight and radii of the disk, we get the moment of inertia of the disk(I_D)=$\frac{1}{2}MR^2$=16560 $gm - cm^2$

5.3 Measurement of I_R of ring

S.No	Wire len. cm	T.P of ring+disk s	T.P of disk s	I_R $gm - cm^2$
1		6.59	5.21	9934.50
2	30.3	6.59	5.22	9833.08
3		6.62	5.23	9972.18
4		7.60	6.01	9921.26
5	41	7.60	6.03	9745.88
6		7.63	6.04	9866.25
7		8.26	6.53	9936.84
8	49.2	8.25	6.52	9953.86

6 Result

Thus, the mean $I_R = 9895.48\ gm - cm^2$
Now, if M is the mass of a ring, and R_1 and R_2 are the inner and outer radii of the ring, the moment of inertia $I_R = \frac{1}{2}M({R_1}^2 + {R_2}^2)$
So, theoretically, $I_R = 9892.16\ gm - cm^2$
Hence the percentage deviation from the theoretical value is =
$\frac{9895.48-9892.16}{9892.16} \times 100 = 0.034\%$

7 Discussion

a)There can be kinks in the wire due to which the torsional coefficient increases. So, this gives rise to a systematic error.
b)The centre of mass of the disk may not co-incide with the axis of the wire.
c)During the oscillation, there can be damping effects due to which the oscillations may not be uniform. Hence, a number of readings are taken.
d)Theoretically, $\frac{I_R}{I_{RD}} = 0.595$. Here, the value is $\frac{9895.48}{16560} = 0.598$

8 Error Analysis

From the formula for of I_R, we get the error to be

$$\frac{dI_R}{I_R} = 2 \cdot \frac{dT_{RD}}{T_{RD}} + 2 \cdot \frac{dT_D}{T_D} \tag{11}$$

Here, $dT_{RD} = dT_D = 0.01$ s
Also, $T_{RD} = 7.63$ s and $T_D = 6.04$ s
So, $\frac{dI_R}{I_R} = 5.93 \times 10^{-3}$
Hence, percentage error $= \frac{dI_R}{I_R} \times 100 = 0.59$ %
Hence, corrected reading $= (9895 \pm 58)$ $gm - cm^2$