Quantum Mechanics I: End-sem examination

Total: 50 marks

(1) Consider a particle of mass m in a 1-dimensional box. The potential has the form $V(x) = \infty,\ x < 0, x > c,\quad V(x) = V_0,\ a < x < b,\quad V(x) = 0,\ 0 < x < a, b < x < c$. Find the quantization condition for stationary state energy levels for energies $E > V_0$, and the corresponding wave-function. In the limit of E much larger than V_0, show that your resulting wave-function and quantization condition reduce to the familiar expressions for a particle in a box with no internal "bump" (i.e. with $V_0 = 0$). [11 mks]

(2) A photon is emitted when a hydrogen atom transits between neighbouring stationary energy levels $n+1$ and n. What is the minimum de Broglie wavelength of the electrons emitted when such photons fall on a metal of workfunction W_0 if the photon energy is known to be above the threshold ? If $W_0 = 3$ eV, what is the maximum n for which this phenomenon is possible ? [5 mks]

(3) A particle of mass m in a box (potential $V(x) = \infty, x < 0, x > a$ and $V(x) = 0, 0 < x < a$) is initially (at $t = 0$) in a state $\psi(x) = 1,\ x < \frac{a}{2},\quad \psi(x) = 0,\ x > \frac{a}{2}$. What is the coefficient of the energy level n of the wavefunction ? Use the normalized stationary wavefunctions $\sqrt{\frac{2}{a}}\ \sin(\frac{n\pi x}{a})$. Write down the wavefunction for the time evolved state at a later time t. [11 mks]

(4) Using the angular momentum algebra $[L_x, L_y] = i\hbar L_z$, etc, and the eigenstates $L^2|l,m\rangle = \hbar^2 l(l+1)|l,m\rangle,\ L_z|l,m\rangle = m\hbar|l,m\rangle$, find the coefficient c^+_{lm} in $L_+|l,m\rangle = c^+_{lm}|l,m+1\rangle$, where $L_+ = L_x + iL_y$ is the raising operator. [7 mks]

(5) Recall the definitions $a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}(x - \frac{ip}{m\omega})$ and its Hermitian conjugate, satisfying $[a, a^\dagger] = 1$, and $a|n\rangle = \sqrt{n}|n-1\rangle,\ a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$, for the harmonic oscillator: in the Schrodinger position space representation with $p = -i\hbar\partial_x$, the $a, a^\dagger$ are differential

operators. Derive the functional form of the ground state wavefunction $\psi_0(x)$. Then evaluate the position space wavefunction corresponding to the excited state $|3\rangle$ to find the functional form of the third Hermite polynomial $H_3(x)$. [10 mks]

(6) Consider a potential $V(x) = V_0, \; x < 0, \; V(x) = 0, \; x > 0$. Find the reflection and transmission coefficients for a particle of mass m coming in from the left $x = -\infty$. [6 mks]