Mathematical Physics I, 2008
Assignment 3

1. Compute the line integral,

$$I = \int_C \vec{A}(\vec{x}) \cdot d\vec{x} \tag{1}$$

where C is a curve parametrised by $\vec{x}(s),\ 0 \leq s \leq 1$, for

(a)

$$\vec{x}(s) = R\left(\cos(2\pi s)\hat{x} + \sin(2\pi s)\hat{y}\right) \tag{2}$$

$$\vec{A}(\vec{x}) = \phi\left(\frac{-y}{\sqrt{x^2+y^2}}\hat{x} + \frac{x}{\sqrt{x^2+y^2}}\hat{y}\right) \tag{3}$$

(b)

$$\vec{x}(s) = R\left(\cos(2\pi s)\hat{x} + \sin(2\pi s)\hat{y}\right) \tag{4}$$

$$\vec{A}(\vec{x}) = -By\hat{x} \tag{5}$$

(c)

$$\vec{x}(s) = a\,\cos(2\pi s)\hat{x} + b\,\sin(2\pi s)\hat{y} \tag{6}$$

$$\vec{A}(\vec{x}) = -By\hat{x} \tag{7}$$

(d)

$$\vec{x}(s) = \frac{R}{s}\hat{z} \tag{8}$$

$$\vec{A}(\vec{x}) = \kappa\frac{\vec{x}}{(x^2+y^2+z^2)^{\frac{3}{2}}} \tag{9}$$

(e)

$$\vec{x}(s) = a\left(\cos(2\pi s)\hat{x} + \sin(2\pi s)\hat{y}\right) + \frac{R}{s}\hat{z} \tag{10}$$

$$\vec{A}(\vec{x}) = \kappa\frac{\vec{x}}{(x^2+y^2+z^2)^{\frac{3}{2}}} \tag{11}$$

2. Compute the surface integral,

$$I = \int_\Sigma \vec{A}(\vec{x}) \cdot d\vec{S} \tag{12}$$

where Σ is a surface parametrised by $\vec{x}(s,t),\ 0 \leq s,t \leq 1$, for

(a)

$$\begin{aligned} \vec{x}(s) &= a\left(s\hat{x} + t\hat{y}\right) + b\hat{z} && (13)\\ \vec{A}(\vec{x}) &= \kappa \frac{\vec{x}}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} && (14) \end{aligned}$$

(b)

$$\begin{aligned} \vec{x}(s) &= as\left(\cos(2\pi t)\hat{x} + \sin(2\pi t)\hat{y}\right) + b\hat{z} && (15)\\ \vec{A}(\vec{x}) &= v_1\hat{x} + v_2\hat{y} + v_3\hat{z} && (16) \end{aligned}$$

(c)

$$\begin{aligned} \vec{x}(s) &= s\left(\cos(2\pi t)\hat{x} + \sin(2\pi t)\hat{y}\right) + \sqrt{1-s^2}\hat{z} && (17)\\ \vec{A}(\vec{x}) &= v_1\hat{x} + v_2\hat{y} + v_3\hat{z} && (18) \end{aligned}$$

(d)

$$\begin{aligned} \vec{x}(s) &= s\left(\cos(2\pi t)\hat{x} + \sin(2\pi t)\hat{y}\right) + \sqrt{1-s^2}\hat{z} && (19)\\ \vec{A}(\vec{x}) &= \kappa \frac{\vec{x}}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} && (20) \end{aligned}$$