# Mathematical Physics I, 2008
## Assignment 2

1. Prove the following identities:

$$\sum_{k=1}^{3} \epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} \quad (1)$$

$$\vec{A} \times \vec{B} \times \vec{C} = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right) \quad (2)$$

$$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{C} \times \vec{D}\right) = \left(\vec{A} \cdot \vec{C}\right)\left(\vec{B} \cdot \vec{D}\right) - \left(\vec{A} \cdot \vec{D}\right)\left(\vec{B} \cdot \vec{C}\right) \quad (3)$$

2. A curve is defined by,

$$\vec{x}(t) = \vec{x}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad (4)$$

   (a) Prove that this curve lies in a plane.

   (b) Compute the normal to this plane.

   (c) Prove that a plane can always be specified by the equation:

$$\vec{a}.\vec{x} = 1 \quad (5)$$

   Find the vector $\vec{a}$ for the plane containing the curve $\vec{x}(t)$

   (d) Choose a convenient coordinate system and show that the curve is a parabola.

3. Consider the surface specified by the equation,

$$z = e^{-\frac{(x^2+y^2)}{2a^2}} \quad (6)$$

   (a) Compute the tangent planes at $x = y = 0$, $x = a, y = 0$ and $x = 4a, y = 0$

   (b) At a given distance, $r$ ($r = \sqrt{x^2 + y^2}$), from the origin find the direction of the steepest slope.

   (c) Find the distance $r$ where the slope is maximum and compute it.

4. Consider the vector field,

$$\vec{A}(\vec{x}) = a(r)\vec{x}, \qquad r = \sqrt{\vec{x}\cdot\vec{x}} \tag{7}$$

   (a) Compute $\vec{\nabla}\cdot\vec{A}$ and $\vec{\nabla}\times A$.

   (b) Find the function $a(r)$ for which the divergence vanishes everywhere except at the origin. Interpret this result in terms of the density of the lines of force.

5. Consider the vector field,

$$\vec{A}(\vec{x}) = a(r)\vec{x}\times\hat{z}, \qquad r = \sqrt{\vec{x}\cdot\vec{x}} \tag{8}$$

   (a) Compute $\vec{\nabla}\cdot\vec{A}$ and $\vec{\nabla}\times A$.

   (b) Find the function $a(r)$ for which the curl vanishes everywhere except at the origin. Interpret this result in terms of the density of the lines of force.