PROPOSAL FOR AN EXPERIMENT

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ABSTRACT. In the following document I will give a description of the proposed experiment.

1. The motivation

The experiment that I am proposing is motivated from the following events that I observed :

- Primarily I was influenced by the way paper boats float and move in water when after letting them into the water we start disturbing the water a short distance away. My observation is that the paper boats don't move about in any predictable way even if the disturbance in water is systematic and random effects like wind is absent.
- Secondly the movement of buoys in water has also been a source of fascination for me as to how they bounce about in response to the waves in water.

In all these phenomeon the basic structural features are:

- A finite cavity which contains the water.
- An object which floats on water.
- The relative size between the float and the cavity:
  - The cavity can either be very large compared to the float such that the boundary is practically at infinity and hence doesn't affect the float's motion. This is the case of the buoy floating in the sea.
  - The cavity is not very large compared to the float and hence the boundary geometry has effects on the motion of the float. This is the case of the paper-boat in the mud pool.
- A perturbing force that causes disturbances in the water. But this disturbing force is not strong enough to cause any disturbance more than surface waves. In most cases the disturbance is somehwat periodic.
- Either the float is heavy enough that the effect of wind and such random effects are not influential or such random effects are absent.

In this experiment the basic aim is to study the trajectories of the float (under variations of the variables identified below) in such kinds of motion in a somewhat idealized and scaled down scenarios. The expectation is that inspite of the scaling down process and the idealization the essential geometric properties of the trajectories will be evident.

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2. The rough sketch of the experiment

The various components of the experiment are:

- In the experiment idea is to have some trays made of sizes which can be put on the top of standard laboratory tables. These trays will act as the cavities.
- These trays will be filled with some fluid and a float will be let on them.
- By some mechanism a periodic disturbance will be created on the water. Call that mechanism the **vibrator**.
- By some mechanism the float's 3-dimensional motion will be tracked. The float will have a 3-dimensional motion since not only will it move on the surface of the water but it will also bounce vertically.

3. The variables in the experiment

In this experiment I identfy the following variables:

(At this point I note as many parameters that I can identify without regard to their relative importance. Later some of the factors may be found to be insignificant or unnecessary and then I will ignore it)

(1) **Geometry of the boundary of the tray** (calling it ∂T)
(2) **Depth of the fluid in the tray** (calling it h)
(3) **Density of the fluid** (calling it ρ)
(4) **Viscosity of the fluid** (calling it σ)
(5) **Frequency of the vibrator** (calling it ν)
(6) **Amplitude of the vibrator** (calling it A)
(7) **Surface area of contact of the vibrator with the fluid surface** (calling it c)
(8) **Mass and volume of the float** (calling it m and V respectively)
(9) **The initial distance of the float from the vibrator** (calling it $d1$)
(10) **Initially the shortest distance between the float and the boundary** (calling it $d2$)
(11) **Temperature of the fluid and the environment** (calling it $T1$ and $T2$ respectively)
(12) **Wind conditions above the water**

Some points to be noted about the variables identified;

- While defining the amplitude of the vibrator we take its mean position to be the surface of the fluid. Hence A effectively measures how much the vibrator dips into the water.
- In defining $d2$ the tacit assumption is that of all the reflected waves from the boundary the ones coming from the nearest points will be most influential.
- For reasons to be discussed below water doesn't seem to be a suitable fluid for the experiment and hence we try to characterize the fluid by its density and viscosity.

4. Implementation of the boundary

There are 2 ways in which the boundary geometry can be changed:

- Separate trays are arranged with different boundary shapes.
- A single tray is had with an aluminium strip in it such that it can be bent into any required geometry as and when needed. (Ofcourse ensuring that the height of the aluminium strip is greater than the depth of the fluid).

The competition between these 2 methods is on the following counts:

- In the first method the boundary can be gotten as rigid and stable as one wants and it can be kept very smooth to ensure that when waves hit it. as low diffraction occurs as possible . But this technique is not versatile since we need separate trays for every ∂T desired.
- In the second method the versatility is attained but we lose out on stability and smoothness of the boundary. (Aluminium sheet is going to develop permanent deformations in the process of changing geometries)

If the need is accuracy then the first method can be opted and if the need is to get a qualitative feel through fast experiments then the second method can be opted.

5. Tracking the float

Since the ultimate aim of the experiment is to study the trajectories in response to the vibrator's parameters and ∂T, it is very crucial to devise ways of tracking the float's motion in water. Following are a few suggestions to track the float and they have been grouped into the following categories:

5.1. Two relatively sophisticated methods.

- One can set up 3 cameras along 3 orthogonal directions and keep taking video footage of the motion and keep some prominently visible object say O, inside the fluid and fixed to the walls. The cameras should be set such that each of the cameras always have O in their field of view.

  Hence effectively we have a video of how the projection of the float varies along 3 mutually orthogonal planes which can be consistently defined taking O as the origin. Now there are video analysis programs available where the programs can track a certain set of pixels across the video footage. Hence these programs can be given the 3 video footages and consistently coordinate systems can be set up for each of the videos with O as the origin for each footage. Hence we have 3 trajectories.

  The information from the 3 analysis can be combined together to generate the trajectory in the 3-Dimensional space.

  For the ease of tracking one can ensure that the float is painted in one single bright colour and hence the programs will need to track pixels of a single colour against a dull background. But ofcourse this method is possible only if the fluid is transparent enough to give enough visibility of the float and O in it.

- We can arrange so that the float has a radio-frequency emitter embedded in it and an external detector to detect it. So from the direction of the radio frequency one can keep calculating the position of the float. But this can get very messy because of problems of the relevant radio wave getting contaminated by other extranneous waves and effects of diffraction.

5.2. **Relatively cruder methods.** One can have a laminated graph paper attached to the base of the tray so that as the object moves the experimenter keeps looking down on it from above and keeps noting down the coordinates over which the float passes. This has the following limitations:

- The accuracy is highly limited by the alertness of the experimenter.
- The data can be obtained only along one of the planes since the boundary geometry is generally not flat and hence one can't do this method of attaching graph papers to the sides.

6. Qualitative understanding of the relative ranges of variables

We note the following interdependancies between the various parameters of the experiment.

- I plan to use three or four different kinds of ∂T for the crates and try to see how the trajectories are affected by it. I plan to use trays such that ∂T = rectangle, triangle, circle and ellipse.
- Choice of h is predominantly affected by A. If the amplitude of the vibrator is large and the depth of the tray is small enough for deep water waves to be set up then we can no more work under the approximation of surface waves. h and A should be so chosen so that the wave disturbance is restricted to the surface.

  In certain approximations of small amplitude surface waves one can show that the amplitude of the disturbance exponentially falls with depth. So depth has to be so chosen that this exponential fall has taken place and the waves reflected from the base of the tank can be ignored.
- It will be helpful to have the viscosity of the fluid low so that the drag forces on the float are minimized and hence we have one less difficult parameter to keep track of.

- We want the waves to take a reasonable amount of time to propagate from the vibrator to the float and for the reflected wave to travel from the boundary to the float.
  - This time interval should not be too high so as to either impractically slow down the experiment or give enough time for the wave to die down by the time it reaches the float. Obviously there are numerous processes by which the the wave's energy can get dissipated while travelling and due to the constraint on h we can't afford increase A as a method to lessen the of dissipation.
  - This time cannot be too short since then coupled with the fact that the dimensions of the box can't be very large (since it is designed to be an indoor table-top experiment) there will be the following problems:
    * Higher wave speed will imply that too many waves will be propagating in the tray simultaneously. Occurrence of too many number of interferences will make the experiment difficult to control.
    * Waves being fast will also require us to have very sensitive motion detectors which is difficult to obtain. This will make timing of the external perturbations difficult and also observation will be difficult.
- The above control on the wave speed and hence on the frequency of interferences happening can be achieved by controlling the following parameters together: $d1$, $d2$, ρ , ν and c .

  Qualitatively these parameters have to be adjusted according to the following considerations:
  - $d1$ and $d2$ determine the distances the waves will travel and hence if they are kept too low we will have uncontrollable amount of interferences happening. c being large will effectively reduce $d1$.
  - ν being large will again lead to fast waves and hence ν can't be very high. But on te other hand if the σ is high then a low ν will increase the drag force. Hence we have more reasons to keep σ low.
  - Low values of ρ will raise the wave speeds and hence ρ must be kept high.
- The m and the V have to be chosen with the following considerations:
  - m and V should be such that the density of the float is lower than that of the fluid. So that it floats.
  - The density shouldn't be too less since that will imply that only a very small fraction of the float is submerged in the fluid and that will make the float very unstable and add extraneous motions to the float that will be difficult to track.
  - The m shouldn't be too low that its motion gets visibly affected by extraneous winds around. It should be high enough such that only the fluid's effect is significant.
  - m can't be kept too high since that will need the V to increase which can't be had since the experiment is supposed to be a table-top experiment.Hence we must take efforts to reduce stray winds and disturbances.
- The experiment will be done indoors and there will be A.C in that room hence temperature doesn't seem to be a big factor in the experiment. The apparatus will never be exposed to any extremes of temperature.

7. Quantitative values for the variables

The system here is highly non-linear given that even the basic equations for the fluid without any external perturbations is the Navier-Stoke's Equation. Hence we try to make some *reasonable* approximations like the following:

- Most primarily we try to go for as less viscous fluid as possible since for viscous fluids approximating any of the variables is incredibly difficult given that they are governed by the highly non-linear Navier-Stokes's equations.
- Velocity of propagation of the wave v is estimated to be $\propto \frac{1}{\sqrt{\rho}}$.
- Amplitude of the wave is estimated to be $\propto A$. Hence once the vibrator's amplitude is fixed we have a limit on the dimensions of the float since it will be convenient to have the typical height of the float to be less than the amplitude of the wave so that waves don't run over the float. This lower bounds V as $V^{\frac{1}{3}} \gg A$ and upper-bound on V is implicit through the size of the tray used. m of the float also gets estimated for a given ρ since it is wanted that $\frac{m}{V} < \rho$.
- ν effectively gives us how many waves are being produced per unit time. So let a frequency of ν translate into number of waves produced per unit time say n. So we can think of the vibrator to be producing n waves in every unit time.
- Hence at any time t there are two kinds of waves reaching the float the ones that are directly reaching it after travelling a distance $d1$ and the ones that are hitting it after refelction from the walls i.e after travelling a distance of $d1 + d2$. Once the fuid is fixed we have a rough estimate of the velocity of propagation of waves in it and hence hence we can say that at any time t there are $2n$ waves hitting the float which can be thought of as 2 groups of n waves apparoaching each other and the 2 groups are distinguished as:
  - n of them were produced at $t - \frac{d1}{v}$ and have come directly to the float.
  - n of them were produced at $t - \frac{d1+d2}{v}$ and have come to the float after reflection.

We further note the following:

- It has to be ensured that the boundary is smooth so that no diffraction of waves happen at it so that if n waves hit it then n waves are reflected from it.
- The first group of n waves are travelling in the same direction and the other n are travelling in direction opposite to it. Hence at the float the number of primary interferences happenning is of the order of n^2. For crude estimates we ignore the secondary interferences where the interferences will occur between the resultant waves from the primary waves.
- Hence crudely we can say that given an n for every unit time n^2 primary interferences happen at the float with intervals of $\frac{d2}{v}$.
- This determines how the basic parameters n, $d2$ and v are going to be set according to the comfort of the experimenter so that he/she doesn't have large number of waves interefering in too frequent batches. The point to note is that:
  - The number of interferences in each batch grow *quadratically* with the ν.
  - The time gap between the batches of waves grows as $\sim d2\sqrt{\rho}$.

Hence the above estimates give a rough idea of how deciding upon the frequency and the complexity of the interferences at the float effectively fix the order of magnitudes of all the basic variables.

We note that $d1$ is essentially a free parameter which has to be set according to the need of smoothness of the wave propagation with the following thing kept in sight:

For a given tray, a too low $d1$ or equivalently a large c might lead to turbulences in the system which are unwanted

8. Final Algorithm to decide the parameters

Hence the above discussion boils down to the realization that before setting the apparatus one must decide on the following depending on one's space and sensitivity constraints and that fixes most of the basic parameters:

- n^2 (governed by sensitivity constraints and it will fix ν)
- $\frac{d2}{v}$ (governed by sensitivity constraints and it will fix $d2$ and ρ)
- m and V (governed by ρ and will fix A)
- $d1$ and c (governed by space constraints and need to avoid turbulence)

9. Final Remarks

Some concluding remarks about the experiment:

- At this level it is difficult to make much of any predictions about the nature of the trajectories obtained but from experience of watching the motion of floating objects in water it seems that there is a possibility of getting interesting curves.
- The only costly apparatus to be used in the experiment are the 3 cameras to track the motion and the video-analysis software. But these can be hired and not permanently bought and hence the experiment is economically viable. The total cost of all the other things, the trays, the fluid and the motor required can be safely put to be below $1000Rs$.
- The experiment has practical relevance as being a scaled down version of real life needs where one needs to understand the motion of various floating objects like buoys and ships and other water tranport. It is important to understand the stability of such floating objects under the influence of continuous perturbation of the waves.