Short sketch of the talk on Aharanov Bohm effect

Non-relativistic treatment.

$$L = \frac{1}{2}mv^2 - q\phi + q\bar{v}\cdot\bar{A}$$

$$A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)$$

$$\bar{p} = \frac{\partial L}{\partial \bar{v}} = m\vec{v} + q\vec{A}$$

$$x^\mu = (ct, x, y, z)$$

$$\therefore H = \bar{v}\cdot\frac{\partial L}{\partial \bar{v}} - L = \frac{1}{2}mv^2 + q\phi$$

$$(\bar{p} - q\bar{A})^2 = m^2v^2$$

$$\therefore H = \frac{1}{2m}(\bar{p} - q\bar{A})^2 + q\phi \quad \text{---(i)}$$

Lorentz force is: $F = q[\bar{E} + \bar{v}\times\bar{B}]$ ---- (ii)

N.B

↳ classical velocity one measures is $\dot{x}$

i.e. $\bar{v} = \frac{\bar{p} - q\bar{A}}{m}$ $\Rightarrow$ $m\bar{v} = \bar{p} - q\bar{A}$

& not $\frac{\bar{p}}{m}$

↳ We call $\bar{p}$ the "Canonical Momentum"

$m\bar{v}$ the 'classical Momentum'

↳ $A'_\mu = A_\mu - \frac{\partial\psi}{\partial x^\mu}$

$$\vec{A}' = \vec{A} + \nabla\psi$$

$$\frac{\phi'}{c} = \frac{\phi}{c} - \frac{1}{c}\frac{\partial\psi}{\partial t}$$

which leaves $E = -\nabla\phi - \frac{1}{c}\frac{\partial A}{\partial t}$ & $B = \bar{\nabla}\times\bar{A}$ invariant

↳ $\dot{p} = -\frac{\partial H}{\partial q}$, $\dot{q} = \frac{\partial H}{\partial p}$

In general $\bar{p}$ is NOT gauge invariant

but $m\bar{v}$ is

$$H = \frac{1}{2m}\left(\bar{p} - q\bar{A}\right)^2 + q\phi$$

~~$\therefore$ the space-component of the Schrodinger eqn. is.~~

$$\cancel{H\psi = E\psi}$$

make the transitions $\vec{p} \rightarrow -i\hbar\bar{\nabla}$

$$H \rightarrow i\hbar\frac{\partial}{\partial t}$$

the space-component of the Schroedinger's eqn. is:

$$\frac{1}{2m}\left(-i\hbar\bar{\nabla} - q\bar{A}\right)^2\psi(x) + q\phi\,\psi(x) = E\,\psi(x) \quad \text{--- (iii)}$$

Let $\psi^0(x)$ be the solution for

$$\frac{1}{2m}\left(-i\hbar\bar{\nabla}\right)^2\psi^0(x) + q\phi\,\psi^0(x) = E\,\psi^0(x) \quad \text{— (iv)}$$

$\therefore$ we guess the solution to (iii) for $\vec{B} = 0$ region.

$$\psi(x) = \psi^0(x)\,e^{\frac{iq}{\hbar}\int_{c}^{s(x)} A(x')\cdot ds'} \quad \text{—— (v)}$$

→ the line integral is over a curve c, that starts at ∞ parametrise by the fuction s such that $s(x)$ is x itself. so the A (at the startpt) = 0

i.e any curve that ends at x.

Lets check that (v) is a solution of (iii)

$$\rightarrow \quad (-i\hbar\nabla - q\bar{A})\psi(x)$$

$$= -q\bar{A}\,\psi(x) - i\hbar\left\{ e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'}\nabla\psi^0(x) + \psi^0(x)\,e^{\frac{iq}{\hbar}\int^{s(x)} A(x')\cdot ds'}\,\frac{iq\,A(x)}{\hbar} \right\}$$

$$= -q\bar{A}\,\psi(x) - i\hbar\,\nabla\psi^0(x)\,e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'} + q\,A(x)\,\psi^0(x)\,e^{\frac{iq}{\hbar}\int^{s(x)} A(x')\cdot ds'}$$

$$= e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'}\left\{ -q\bar{A}(x)\psi^0(x) - i\hbar\nabla\psi^0(x) + q\bar{A}(x)\,\psi^0(x) \right\}$$

$$i\hbar\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'}$$

$$(-i\hbar\nabla - q\bar{A})^2\psi(x) = (-i\hbar\bar{\nabla} - q\bar{A})(-i\hbar)\nabla\psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'}$$

$$= -\hbar^2\nabla\psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'} \quad \frac{iq\,A(x)}{\hbar}$$

$$+\; iq\bar{A}\hbar\,\nabla\psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'}$$

$$= \cancel{-i\hbar q\, A(x)\nabla\psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'}} \cancel{+ iqA\hbar}$$

$$+\; -\hbar^2\left\{\nabla^2\psi^0(x)\right\} e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'}$$

$$\Rightarrow \quad (-i\hbar\bar{\nabla} - q\bar{A})^2\psi(x) = -\hbar^2 e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'}\, \nabla^2\psi^0(x) \quad \text{--- (vi)}$$

$\therefore$ (iv) implies:

$$-\frac{\hbar^2}{2m}\nabla^2\psi^0(x) + q\phi\,\psi^0(x) = E\,\psi^0(x)$$

$\therefore$ substitute in (vi) it means,

$$(-i\hbar\nabla - q\bar{A})^2\psi(x) = e^{\frac{iq}{\hbar}\int^{s(x)} A(x').ds'} \{2m\}\left(E\psi^0(x) - q\phi\,\psi^0(x)\right)$$

$$\Rightarrow \quad \frac{1}{2m}(-i\hbar\bar{\nabla} - q\bar{A})^2\psi(x) + q\phi\,\psi(x) = E\,\psi(x) \ldots\ldots$$

the proof.

N.B An important observation:-

$$\psi(x) = \psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} A(x')ds'} \qquad \{\bar{B} = 0,\ \bar{A} \neq 0\}$$

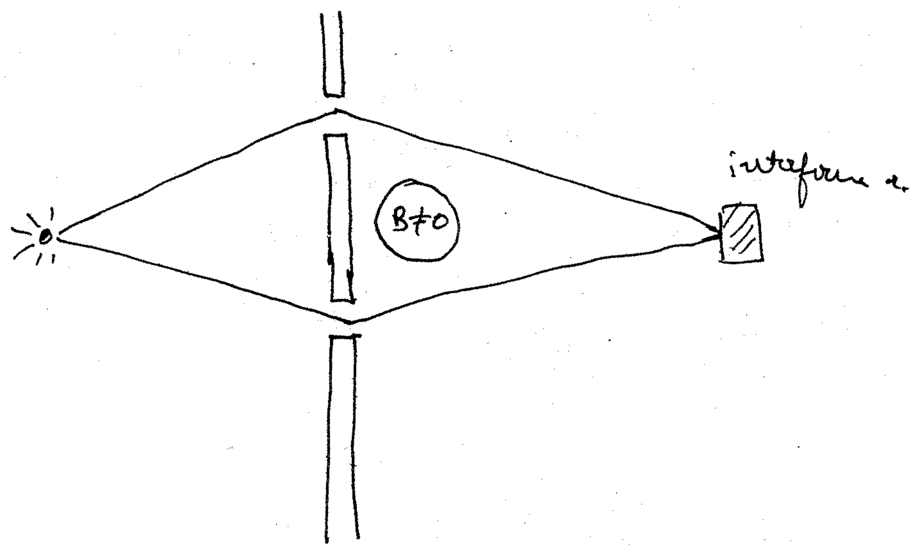
$$A' = A + \nabla X$$

$$\psi'(x) = \psi^0(x)\, e^{\frac{iq}{\hbar}\int^{s(x)} \nabla X.ds'}$$

$$\Rightarrow \quad \psi'(x) = \psi(x)\, e^{\frac{iqX}{\hbar}}$$

... is a change of phase of

Ahranov Bohm Effect :



$$\Psi_{\text{at the interference region}} = \psi_1^{(0)} e^{\frac{iq}{\hbar}\int_{\text{Path 1}}^{s(x)} A(x').ds'} + \psi_2^{(0)} e^{\frac{iq}{\hbar}\int_{\text{Path 2}}^{s(x)} A(x').ds'}$$

$$\Big\{$$

$$\cos\left(\frac{q}{\hbar}\int_1^{s(x)} A(x')\,ds'\right) + \cancel{\cos}\left(\frac{q}{\hbar}\int_2^{s(x)} A(x').ds'\right)$$

$a \qquad b$

$|\Psi_{\text{an}}|^2$

$$\sim \cos\left(\frac{1}{2}\frac{q}{\hbar}\left\{\int_1^{s(x)} A(x').ds' - \int_2^{s(x)} A(x')\,ds\right\}\right)$$

$= \left|2\cos\frac{a+b}{2}\cos\frac{a-b}{2} + i\,2\sin\frac{a+b}{2}\cos\frac{a-b}{2}\right|$

$2 \ \& \ \cos^2\frac{a-b}{2}$

$$\sim \cos\left(\frac{1}{2}\frac{q}{\hbar}\oint A.\,d\vec{s}\right)$$

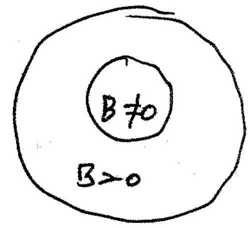
$$\cos\left(\frac{q}{2\hbar}\phi\right)$$

So there is an interference effect: due to the [illegible] cylinder.

Quantization of magnetic flux in Superconductor.

$\downarrow$

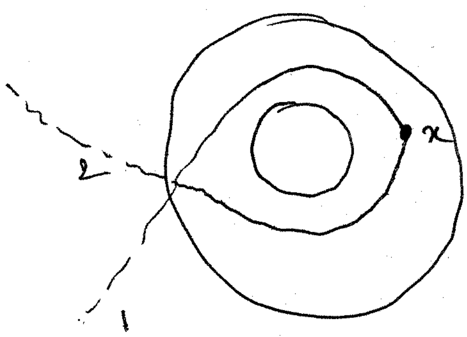
is a perfect diamagnet

$\rightarrow$ 

$\rightarrow$ Meissner effect.

$\rightarrow$ Coupling — so we use the charge as $2e$

quantum-mechanical "quasi-particle".



$x = s(x)$

$$\psi(x) = \psi^0 e^{\frac{2ei}{\hbar}\int^{s(x)} A(x').ds'}$$

$$\psi^0 e^{\frac{2ei}{\hbar}\int_1^{s(x)} A(x').ds'} = \psi^0 e^{\frac{2ei}{\hbar}\int_2^{s(x)} A(x').ds'}$$

so on the

$$\Rightarrow e^{\frac{2ei}{\hbar}\oint A.ds} = 1$$

$$\Rightarrow e^{\frac{2ei\phi}{\hbar}} = 1$$

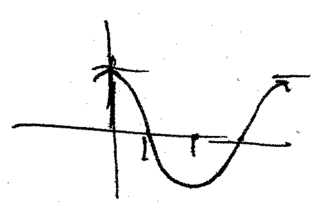
$e^{i\theta} = \cos\theta + i\sin\theta = 0$

$\cos 2n\pi + i\sin 2n\pi$

$e^{i2n\pi} = 1$

$\therefore \frac{2e\phi}{\hbar} = 2n\pi$

$\Rightarrow \phi = \frac{n\pi\hbar}{e}$



$\therefore \phi$ increases in steps of

$$\frac{\pi\hbar}{e} = \frac{\pi h}{2e}$$

$\rightarrow$ F-London, Onsager (Confirm with the fact 2)

$\rightarrow$ 1961 by B.S Deaver & W.H Fairbank.

$\rightarrow$ R. Doll & Nabauer.

} Confirmed