

- You shall receive feedback on the problems *only if*:
    1. You submit to Ankita by **2359 hrs on Thursday, October 17, 2019**, and
    2. **Submit each problem in a separate sheet** with your name on each sheet. This is essential because the TAs divide correction duties by problem.
  - This problem set should take you approximately an hour to solve. This is the pace that will be expected in the quizzes.
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*“You only grow by coming to the end of something  
and by beginning something else.”* – John Irving

1. Let  $G$  be a CFG in Chomsky normal form.
    - (a) Show that for any string  $w \in L(G)$  of length  $n > 1$ , exactly  $2n - 1$  steps are required for any derivation of  $w$ .
    - (b) Let  $G$  contain  $b$  non-terminals. Show that, if  $G$  generates some string with a derivation having at least  $2b$  steps, then  $L(G)$  is infinite.
  2. Recall the *shuffle* operator from Problem Set 3. Prove or disprove:
    - (a) Given  $L$  context-free and  $R$  regular,  $\text{shuffle}(L, R)$  is context-free.
    - (b) Given  $L_1, L_2$  context-free,  $\text{shuffle}(L_1, L_2)$  is context-free.
  3. Given a CFL  $L$  in the form of a CFG or a PDA, how would you check if  $L$  is finite?
  4. Say that a language is prefix-closed if the prefix of any string in the language is also in the language. Let  $L$  be an infinite, prefix-closed, context-free language. Show that  $L$  contains an infinite regular subset.
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