- **You shall receive feedback on the problems *only if*:**
  1. You submit **to Ankita** by **2359 hrs on Thursday, September 12**, 2019, and
  2. **Submit each problem in a separate sheet** with your name on each sheet. This is essential because the TAs divide correction duties by problem.
- This problem set should take you approximately an hour to solve. This is the pace that will be expected in the quizzes.

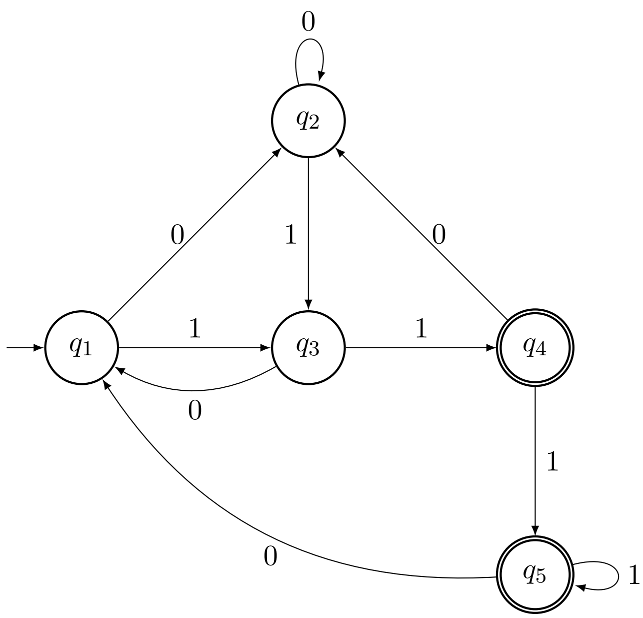
"Nothing in life is to be feared. It is only to be understood." – Marie Curie

1. For languages L_1, L_2 over Σ, define

$$L_1 \setminus |L_2| = \{w \mid \exists x, wx \in L_1, \exists u \in L_2, |u| = |x|\}$$

   If L_1, L_2 are regular, what can you say about $L_1 \setminus |L_2|$?

2. Minimize the following DFA and argue why your construction is correct.



3. We know that regular languages are closed under homomorphism and inverse homomorphism. Now consider the following. $h : \Sigma^* \to \Gamma^*$ be a homomorphism. If, for some $L \subseteq \Sigma^*$, $h(L) \subseteq \Gamma^*$ is regular, then is L regular?

4. We know that for some regular language, there is a unique corresponding DFA with minimal number of states. Is this also true for NFA? That is, if we fix a regular language L, and want to obtain an NFA accepting L with the smallest possible number of states, is there exactly one such NFA?

5. ***This problem is not for submission.*** *It is harder than the rest and it is fine if you cannot complete it within the time limit. We will discuss it in the next tutorial.*

   We call $k \in \mathbb{N}$ a period of a language $L \subset \{a\}^*$ (language over a unary alphabet) if $a^n \in L$ implies $a^{n+k} \in L$ for large enough n. For $L \subset \{a\}^*$, prove that the following are equivalent:

(a) L is regular

(b) L can be represented as a finite union of languages of the form

$$L_{a,d} = \{a^n \mid n = a + kd, k \in \mathbb{N}\}$$

(c) $\exists! t \in \mathbb{N}$ such that k is a period of L if and only if it is an integral multiple of t.