

- You shall receive feedback on these problems *only if*:
 1. You submit to Ankita by **2359 hrs on Thursday, August 29**, 2019, and
 2. **Submit each problem in a separate sheet** with your name on each sheet. This is essential because the TAs divide correction duties by problem.
- This problem set should take you approximately an hour to solve. This is the pace that will be expected in the quizzes.

“...one man in his time plays many parts, his acts being seven ages.” – William Shakespeare

1. The *Nerode automaton* for a regular language is the automaton such that its states are the quotient languages. Give the Nerode automaton for the following rational expression over $\Sigma = \{a, b\}$:

$$a^*b^* + b^*a^*$$

2. Let $\Sigma = \{a\}$. Is the language

$$L = \{a^m \mid m = 2^n, n \geq 0\}$$

regular? Justify.

3. For $n \in \mathbb{N}$, let $F(n)$ be the n^{th} Fibonacci number. Recall that they are defined recursively as :

$$F(1) = 1$$

$$F(2) = 1$$

$$\forall n > 2, F(n) = F(n-1) + F(n-2)$$

For $\Sigma = \{a\}$, consider the language

$$L = \{a^m \mid m = F(n), n > 0\}$$

Is L regular? Justify.

4. Let $\Sigma = \{a_1, \dots, a_n\}, n > 1$. Let S_n be the set of permutations of $\{1, \dots, n\}$. Consider the language

$$L = \{a_{\sigma(1)}^{m_1} \cdots a_{\sigma(n)}^{m_n} \mid \sigma \in S_n, m_1, \dots, m_n \geq 0, m_1 + \cdots + m_n = 2^m, m \geq 0\}$$

Is L regular? Justify.