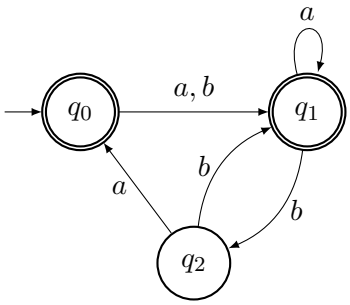
- **You shall receive feedback on these problems *only if***:
  1. You submit **to Ankita** by **2359 hrs on Thursday, August 22**, 2019, and
  2. **Submit each problem in a separate sheet** with your name on each sheet. This is essential because the TAs divide correction duties by problem.
- This problem set should take you approximately an hour to solve. This is the pace that will be expected in the quizzes.

"Express yourself." – Madonna

1. Convert the following automaton to a rational expression:



2. Write a rational expression for the language

$$L = \{w \mid w \text{ does not contain 101 as a substring, } |w| \text{ is even}\}$$

3. Define the *shuffle* of two words w_1, w_2 as follows.

$$\text{shuffle}(w_1, w_2) = \{u_1 v_1 \cdots u_m v_m \mid u_1, \ldots, u_m, v_1, \ldots, v_m \in \Sigma^*, w_1 = u_1 \cdots u_m, w_2 = v_1 \cdots v_m\}$$

Using this, define the *shuffle* of two languages $L_1, L_2 \subseteq \Sigma^*$ as:

$$\text{shuffle}(L_1, L_2) = \{w \in \text{shuffle}(w_1, w_2) \mid w_1 \in L_1, w_2 \in L_2\}$$

Show that if L_1 and L_2 are recognizable, so is $\text{shuffle}(L_1, L_2)$.

4. Define the *middle-third* of a language L as follows:

$$\text{middle-third}(L) = \{v \in \Sigma^* \mid \exists u_1, u_2 \in \Sigma^*, |u_1| = |v| = |u_2|, u_1 v u_2 \in L\}$$

Show that if L is recognizable, $\text{middle-third}(L)$ is also recognizable.
