- **You shall receive feedback on the problems *only if***:
  1. You submit **to Ekanshdeep** by **2359 hrs on Friday, November 22**, 2019, and
  2. **Submit each problem in a separate sheet** with your name on each sheet. This is essential because the TAs divide correction duties by problem.

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1. Given two CFGs G_1 and G_2, show that it is undecidable to check whether $L(G_1) \cap L(G_2) \neq \phi$ by giving a reduction from Post's Correspondence Problem (PCP).

2. Give a queue automaton which accepts the following language:

$$L = \{w\#w^R \mid w \in \{a, b\}^*\}$$

3. In class it was stated that PCP is undecidable. Below is a list of steps by which you can prove this theorem. Follow along and fill in the missing details:

   (a) We introduce Modified PCP as the following problem:
   Given $(w_1, v_1), ..., (w_n, v_n)$, do there exist indices $i_2, ..., i_k$ such that $w_1 w_{i_2} ... w_{i_k} = v_1 v_{i_2} ... v_{i_k}$ (note the fixed first index).

   Reduce MPCP to PCP (Hint: In a given instance of MPCP, what would happen if you added a new symbol to the beginning of w_1 and v_1 and viewed it as an instance of PCP? Will this work? If not, what other similar modifications can you make on the strings?)

   (b) The next step is to reduce halting problem to MPCP. Given an instance of halting problem (M, w), we will construct an instance of MPCP (A, B) which has a solution iff M halts on w.

   - The idea is we will simulate partial computations of M on w in our strings. We begin with the initial tape (note the use of MPCP for this), and ensure that the A-string is always one step behind in computation than the B-string.
   - Once the computation halts, we will allow the A-string to "catch up" and match the B-string; so the MPCP will have a solution iff M halts on w.
     Hint: Let us represent turing machine configurations as uqv, $u, v \in \Sigma^*, q \in Q$, and separate successive configurations by # in the string. In the instance of MPCP, let $w_1 = \#$ and $v_1 = \#q_0 w$. Also note that successive turing machine configurations change only near the head—the rest of the tape remains unchanged.

   Please refer to *Hopcroft, Motwani, Ullman* for a complete proof of undecidability of PCP.
