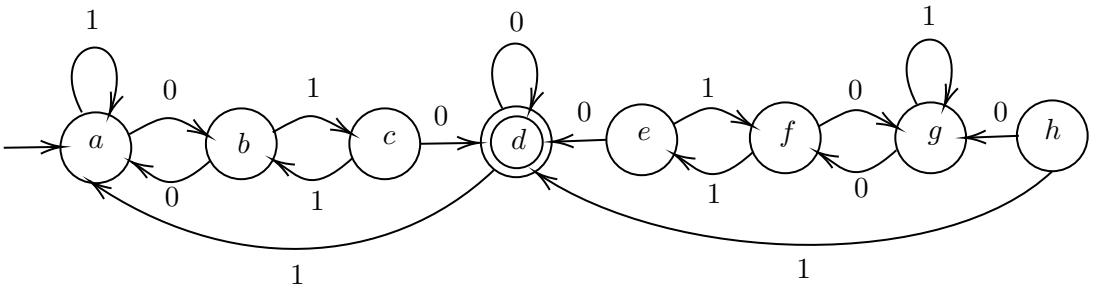
1. Prove whether or not the follwoing languages are regular:
   (a) $L_1 = \{www \mid w \in \{a, b\}^*\}$
   (b) $L_2 = \{a^{2^n} \mid n \geq 0\}$
   (c) $L_3 = \{w \in \{0, 1\}^* \mid w$ has equal occurrences of 01 and 10 as substrings $\}$.
   (d) $|w|_a - |w|_b \leq 8$
   (e) $\{a^{p-1} \mid$ p is prime$\}$
   (f) $\{ucv \mid u, v \in \{0, 1\}^*\}$
2. (a) Give 2 non-regular languages $L_1, L_2 \subseteq \{0, 1\}^*$ for which L_1L_2 is regular.
   (b) If L_1L_2 is regular, is L_2L_1 regular?
   (c) Is L_1 regular, given L_2, L_1L_2, L_2L_1 are regular?
   (d) If L^2 is regular, is L regular?
3. Construct nerode automata for $L = (a^* + b)ab^*$
4. Find the minimum state DFA for the following DFA:



5. Is the language $L = \{w \mid w = uv = vu$ for some $u, v \in \Sigma^* \backslash \{\epsilon\}\}$ regular? If yes, provide the Nerode automaton and a proof of non-regularity otherwise.
6. For a language $L \subseteq \Sigma^*$, define $LOG(L) = \{u \in \Sigma^* \mid \exists u$ s.t. $|v| = 2^{|u|}$ and $uv \in L\}$. Show that if L is regular, so is $LOG(L)$.