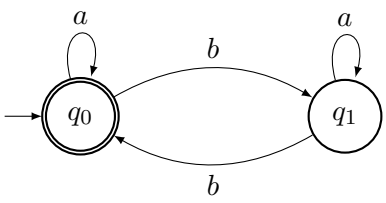
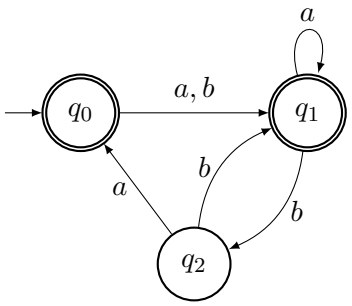
"I always have a quotation for everything – it saves original thinking." – Dorothy L. Sayers

1. Convert the following automata to rational expressions:





2. Convert the following rational expressions to automata:
   (a) $(a + c)^*(b(b + c)^*a(a + c)^*)^*$
   (b) $a^*(b(a^*ba^*)^*b)^*$

3. Give rational expressions for the following languages over $\Sigma = \{0, 1\}$:
   (a) $\{w \mid w$ begins with a 0 and ends with a 1$\}$
   (b) $\{w \mid w$ contains at least three 1s$\}$
   (c) $\{w \mid w$ has length at least 5$\}$
   (d) $\{w \mid w$ does not contain consecutive 1s$\}$

4. Prove the following identities of rational expressions. e, e_1, e_2 are rational expressions over $\Sigma = \{a, b\}$
   (a) $(e^*)^* = e^*$
   (b) $(e_1 + e_2)^* = (e_1^* e_2^*)^*$

5. Given a regular language L, prove that the following languages are also regular:
   (a) $min(L) = \{w \mid w$ is in L, but no proper prefix of w is in $L\}$
   (b) $max(L) = \{w \mid w$ is in L, but $\forall x \neq \epsilon$, $wx \notin L\}$

6. (a) Given a finite automaton A on the alphabet Σ and a word $w \in \Sigma^*$, give an algorithm to check if $w \in \mathcal{L}(A)$? How much time does your algorithm take?
   (b) Given a finite automaton A with k states, give an algorithm to check if $\mathcal{L}(A) = \emptyset$. How much time does your algorithm take?
   (c) Show that if an NFA with k states accepts some word of length k or more, then it accepts infinitely many words.
   (d) Show that if an NFA with k states accepts infinitely many words, then it accepts a word of length between k and $2k$, i.e. w s.t. $k \leq |w| < 2k$.
   (e) Given a finite automaton A with k states, give an algorithm to check if $\mathcal{L}(A)$ is finite? How much time does your algorithm take?