1. Show by giving an example that, if A is an NFA that recognizes language L, swapping the accept and non-accept states in A doesn't necessarily yield an NFA that recognizes the complement of L. Is the class of languages recognized by NFAs closed under complementation?

2. Show that NFAs with only one final state are as powerful as the 'regular' NFAs. Is the same true for DFAs? Given any DFA, can we construct an equivalent DFA which contains only one final state?

3. Give an NFA or an ϵ-NFA for the set of strings over $\{0, 1\}$ such that at least one of the last seven positions is a 1.

4. For languages L_1 and L_2, let the shuffle of L_1 and L_2 be the language:

   $\{w \mid w = u_1 v_1 \cdots u_k v_k,$ where the string $u_1 \cdots u_k \in L_1$ and the string $v_1 \cdots v_k \in L_2$ and each $u_i, v_i \in \Sigma^*\}$

   Show that the class of regular languages is closed under shuffle.

5. A map $h : \Sigma \to \Gamma^*$ defines a homomorphism $h : \Sigma^* \to \Gamma^*$ such that

   $$h(\epsilon) = \epsilon \text{ and}$$

   $$h(uv) = h(u)h(v) \text{ for all } u,\, v \in \Sigma^*,$$

   For example, let $h : \{a, b, c\} \to \{0, 1\}^*$ be given by

   $$h(a) = 1,\ h(b) = 01\ h(c) = 0.$$

   Thus, for instance, we have $h(abbc) = 101010$.

   (a) For $L \subseteq \Sigma^*$, let $h(L) = \{h(w) \mid w \in L\}$. Show that, if L is regular $h(L)$ is also regular.

   (b) For $L' \subseteq \Gamma^*$, let $h^{-1}(L') = \{w \mid h(w) \in L'\}$. Show that, if L' is regular $h^{-1}(L')$ is also regular.

6. Suppose L is a regular language.

   (a) If $w = a_1 a_2 \cdots a_n$ then $w^R = a_n a_{n-1} \cdots a_1$. Show $L^R = \{w^R \mid w \in L\}$ is regular.

   (b) Let $\mathsf{palprefix}(L) = \{w \mid w \cdot w^R \in L\}$. Show that $\mathsf{palprefix}(L)$ is regular.