## QUIZ - 4

## Theory of Computation

10/11/2016

1. [3 marks] A word $w$ is said to be cyclic if there exist $u, v \in \Sigma^{*} \backslash\{\epsilon\}$ such that $u v=w=v u$. Design a Turing machine that accepts an input if and only if it is cyclic.
2. [6 marks/ Consider the following languages. (Here, $\langle M\rangle$ denotes the description of a Turing machine $M$.)

$$
\begin{aligned}
L_{1} & =\{\langle M\rangle \mid M \text { halts on input }\langle M\rangle\} \\
L_{2} & =\{\langle M\rangle \mid M \text { accepts input }\langle M\rangle\}
\end{aligned}
$$

Justify your answers to the following questions.
(a) Is $L_{1}$ recursive? Is it recursively enumerable? Is it co-recursively enumerable?
(b) Is $L_{2}$ recursive? Is it recursively enumerable? Is it co-recursively enumerable?
(c) Is $\left(L_{1}\right)^{c} \cup L_{2}$ recursive? Is it recursively enumerable? Is it co-recursively enumerable?
(d) Is $L_{1} \cup\left(L_{2}\right)^{c}$ recursive? Is it recursively enumerable? Is it co-recursively enumerable?
3. [5 marks] The language-finiteness problem for context-free grammars is given below.

| LANG-FINITENESS |  |
| :---: | :---: |
| Input: | A context-free grammar $G$ |
| Question: | Is $L(G)$ finite? |

Is LANG-FINITENESS decidable? Justify.

