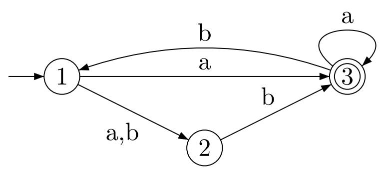
Theory of Computation

Mid-semester Exam — 21/09/2016

Maximum marks: 30. Duration: 3 hours. All questions carry 5 marks.

Rational Expressions

1. Give a rational expression for the language recognized by the following automaton.



Squares and Roots

Let $L \subseteq \Sigma^*$ be a language. We define the languages $\mathsf{root}(L)$ and $\mathsf{square}(L)$ as follows:

$$\mathsf{root}(L) = \{w \mid ww \in L\}$$

$$\mathsf{square}(L) = \{ww \mid w \in L\}$$

2. Suppose L is regular. Should $\mathsf{root}(L)$ be necessarily regular? Justify.
3. Suppose L is regular. Should $\mathsf{square}(L)$ be necessarily regular? Justify.

Subwords - upward and downward closures

Let $u, v \in \Sigma^*$ be two words. We say that u is a subword of v, denoted $u \preceq v$, if u can be obtained from v by deleting some of its letters. That is, $u \preceq v$ if 1) u is of the form $a_1 a_2 \dots a_n$, $a_i \in \Sigma$, $n \geq 0$ and 2) v is of the form $x_0 a_1 x_1 a_2 x_2 \dots x_{n-1} a_n x_n$ where $x_i \in \Sigma^*$ for each $0 \leq i \leq n$.

Let $L \subseteq \Sigma^*$ be a language. The downward closure of L (denoted $\downarrow L$), and upward closure of L (denoted $\uparrow L$) are languages defined as follows:

$$\downarrow L = \{u \mid \exists v \in L, u \preceq v\}$$
$$\uparrow L = \{v \mid \exists u \in L, u \preceq v\}$$

4. Suppose L is regular. Is $\downarrow L$ necessarily regular? Justify.
5. Suppose L is regular. Is $\uparrow L$ necessarily regular? Justify.
6. Suppose L is regular. Is $\downarrow(\mathsf{square}(\uparrow L))$ necessarily regular? Justify.