Problem Set 7

Theory of Computation

October 13, 2017

Some people just want to watch the Turing machine run.

Problem 1. *Let L_1 and L_2 be context free languages. Show that it cannot be decided if $L_1 \cap L_2 = \emptyset$ by reducing PCP to this problem.*

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Problem 2. *Consider the following languages :*

$$L_1 = \{\langle M \rangle : M \text{ accepts } \epsilon\}$$
$$L_2 = \{\langle M \rangle : M \text{ rejects } \epsilon\}$$

Prove that both these languages are undecidable. (Can you use Rice's theorem to do so?)

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Problem 3. *A two-headed finite automaton (2DFA(2)) is a deterministic finite automaton that has two read-only bidirectional heads which can be independently controlled. The tape of the 2DFA(2) contains just the input and two delimiter symbols ($\vdash, \dashv$) on either side of the tape. A 2DFA(2) accepts by entering a special accept state. Formally, a 2DFA(2) is of the form $(Q, \Sigma, \delta, q_0, q_{accept}, q_{reject})$ where*

$$\delta : Q \times (\Sigma \cup \{\vdash, \dashv\}) \times (\Sigma \cup \{\vdash, \dashv\}) \to Q \times \{L, R\} \times \{L, R\}$$

For example we can show that such a machine can accept $\{a^n b^n : n \in \mathbb{N}\}$. Consider the following languages:

$$L_1 = \{\langle M, w \rangle : M \text{ is a 2DFA(2) and } M \text{ accepts } w\}$$
$$L_2 = \{\langle M \rangle : M \text{ is a 2DFA(2) and } L(M) \neq \emptyset\}$$

Show that one of them is decidable and the other one is not.

$\dashv$

Problem 4. *PCP is undecidable over the binary alphabet. What can you say about PCP over a unary alphabet?*

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Problem 5. *Consider a model of computation, where we have a Turing machine but the tape associated with it is a 2-D tape. The 2-D tape corresponds to the upper right quadrant of a plane and the transition functions of the Turing machine enables the head to move left, right, up or down the tape. Show that this model is as powerful as a Turing machine.*

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We're the TAs you deserve, but not the one you need right now.